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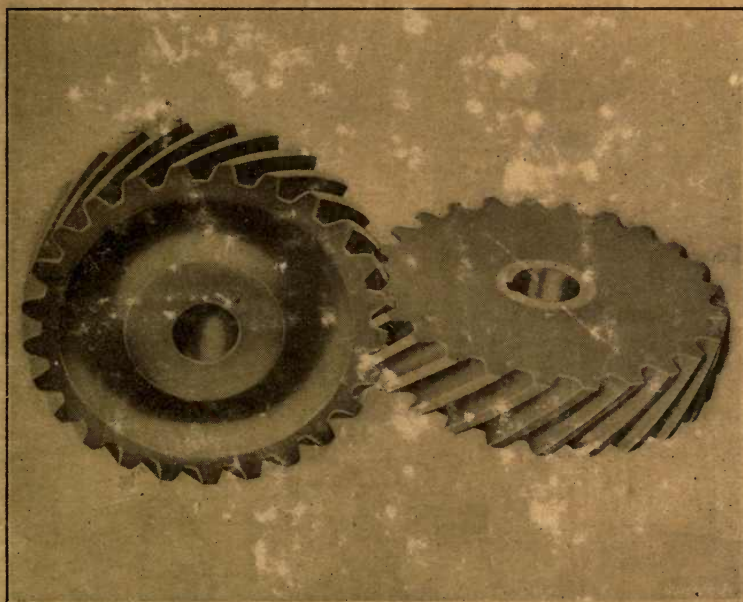
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# SPIRAL GEARING

CALCULATION AND DESIGN—HERRING-  
BONE GEARING—GEAR HOBGING

THIRD EDITION—REVISED AND ENLARGED



MACHINERY'S REFERENCE SERIES—NO. 20  
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# MACHINERY'S REFERENCE SERIES

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MACHINE DESIGN AND SHOP PRACTICE REVISED AND  
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NUMBER 20

## SPIRAL GEARING

THIRD EDITION—REVISED AND ENLARGED

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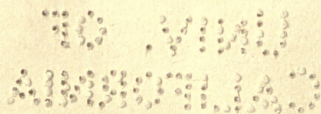
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NUMBER 20  
SPIRAL GEARING

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The present—the third—edition of this number of MACHINERY'S Reference Series has been thoroughly revised, and a considerable amount of new matter has been included. Chapters on "Herringbone Gears" and on "Calculating Gears for Generating Spirals on Hobbing Machines" have been added, and the chapter on "Setting the Table when Milling Spiral Gears" has been entirely rewritten.



## CHAPTER I

### RULES AND FORMULAS FOR DESIGNING SPIRAL GEARS

In accordance with time-honored custom, this contribution to the art of designing helical or "spiral" gears opens with an apology. The subject is one which, from its very nature, can be approached by any one of a number of different ways, and it has been approached by so many of these possible different ways that perhaps the subject has become quite confused in the minds of many readers of technical literature. The writer does not offer the excuse of novelty in the methods presented in the following paragraphs, since some of the details which were independently worked out by him have been described by others. His reason for adding one more to the series of solutions of helical gear problems is that the method described appears to reduce the most serious of this class of problems (Class II, page 9) to its simplest elements. The method of procedure will be described without proof or comment.

The terms "spiral gear" and "helical gear" are, in usage, synonymous, the only difference being that the former of these terms is absolutely incorrect. Inasmuch, however, as the word "spiral" is in such common use among mechanics in this connection, the writer has not had the moral courage to use the more proper term throughout this treatise, but it would be a good plan for the readers to become familiar with the term "helical" as applied to gearing.

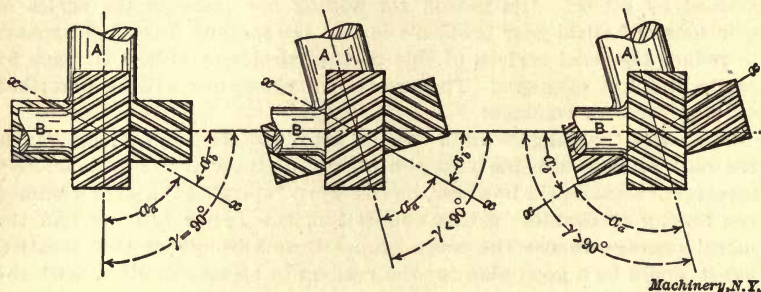
#### Dimensions and Definitions

Some of the terms used will require explanation. The center angle of a pair of helical or spiral gears is the angle made by the two center lines or axes of the gears, as taken in a view perpendicular to both axes. In Fig. 1 are shown views of three sets of spiral gears taken in the plane which shows the center angle. At the left is the ordinary case in which the shafts are at right angles with each other, so that the center angle ( $\gamma$ ) is 90 degrees. In the second case  $\gamma$  is less than 90 degrees, and in the example shown at the right it is more. It should be noted in the last two cases that the position of the shaft axes is identical, but that the two center angles are located on opposite sides of axis  $A$ . In order to know on which side of the center line to take the center angle in cases like those shown, we have to reckon with the position of the teeth of the gears in contact. The center angle is taken at the side which includes the line  $x-x$ , passing lengthwise of the teeth of the gears at the point of contact with each other. Since the teeth are laid out differently in the two cases, the angles are different. The case shown in the center is much the more usual of the two, the other being very rare.

In Fig. 2 is given a diagram showing what is meant by the "tooth angle" of a helical gear. In using the expression "tooth angle," the angle made by the tooth with the axis of the gear is meant, not the angle of the tooth with the face of the gear. Fig. 2 shows  $\alpha_a$  as the tooth angle of gear  $a$ , and  $\alpha_b$  as the tooth angle of gear  $b$ , used in the sense in which we will use them.

The number of teeth and the pitch diameter are terms which are identically the same as those used for spur gearing\* and, therefore, require no explanation. Practically all spiral gears are of small size, and hence are reckoned on the diametral pitch rather than the circular pitch system. All the rules and formulas given will, therefore, make use of the diametral pitch only. This may easily be found from the circular pitch by dividing 3.1416 by the circular pitch. The center distance is, of course, the shortest distance between the axes, and so is measured along the perpendicular common to both of them.

The regular diametral pitch of a spiral gear will be found the same as for a spur gear by dividing the number of teeth by the pitch diam-



Machinery, N.Y.

Fig. 1. Spiral Gears with Different Center Angles

eter in inches. We are not interested in knowing what this is, however, since it does not enter into the calculations at all and since the cutter used has to be for a somewhat finer diametral pitch. This is shown more clearly in Fig. 3. The normal diametral pitch, or diametral pitch of the cutter used, is reckoned from measurements taken along the pitch cylinder at right angles to the length of the tooth.  $P'$  represents the regular circular pitch, while  $P_n'$  represents the normal circular pitch. The diametral pitch may be found from this by dividing 3.1416 by  $P_n'$ . This is the pitch of the cutter to be used. The cutter, as explained on page 5, cannot be selected for the actual number of teeth in the gear, but must take into account the helix angle of the teeth as well, since the curvature as measured on a line at right angles to the helix is at a greater radius than when measured on the circle.

The length of the helix, or the lead, as shown in Fig. 3, is the length of pitch cylinder required to permit one complete revolution of the tooth if the latter were carried around for the full length of this cylinder. In Fig. 4, the relation of lead, circumference, and tooth angle is plainly shown, the helix  $AB$  here being developed on a plane.

\* See MACHINERY'S Reference Series No. 15, Spur Gearing, Chapter II.

The addendum  $S$ , and whole depth  $W$  of the tooth for helical gears is the same as for plain spur gears. The normal thickness of tooth at the pitch line,  $T_n$ , as shown in Fig. 3, is measured in a direction perpendicular to the face of the tooth. The regular tooth thickness is shown at  $T$ , but with this we are not concerned. The outside diameter, as for spur gears, is found by adding twice the addendum to the pitch diameter.

### Rules for Calculating Spiral Gear Dimensions

The following rules are used for calculating the dimensions of spiral or helical gears:

Rule 1. *The sum of the tooth angles of a pair of mating helical gears is equal to the shaft angle; that is to say, in Figs. 1 and 2, angle  $\alpha_a$  added to  $\alpha_b$  equals  $\gamma$ , as is self-evident from the engravings.*

Rule 2. *To find the pitch diameter of a helical gear, divide the num-*

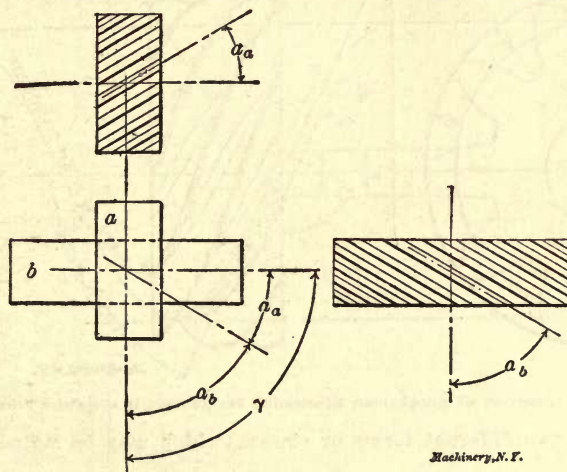


Fig. 2. Diagram Showing Notation used for Tooth Angles

ber of teeth by the product of the normal pitch and the cosine of the tooth angle.

Rule 3. *To find the center distance, add together the pitch diameters of the two gears and divide by 2. This rule is evidently the same as for spur gears.*

Rule 4. *To prove the calculations for pitch diameters and center distance, multiply the number of teeth in the first gear by the tangent of the tooth angle of that gear, and add the number of teeth in the second gear to the product; the sum should equal twice the product of the center distance multiplied by the normal diametral pitch, multiplied by the sine of the tooth angle of the first gear.*

Rule 5. *To find the number of teeth for which to select the cutter, divide the number of teeth in the gear by the cube of the cosine of the tooth angle.*

Rule 6. To find the lead of the tooth helix, multiply the pitch diameter by 3.1416 times the cotangent of the tooth angle.

The rules relating to the addendum and the whole depth of tooth are the same as for spur gears. They are:

Rule 7. To find the addendum, divide 1 by the normal diametral pitch.

Rule 8. To find the whole depth of tooth space, divide 2.157 by the normal diametral pitch.

Rule 9. To find the normal tooth thickness at the pitch line, divide 1.571 by the normal diametral pitch.

Rule 10. To find the outside diameter, add twice the addendum to the pitch diameter.

The problem of designing a pair of spiral gears presents itself in

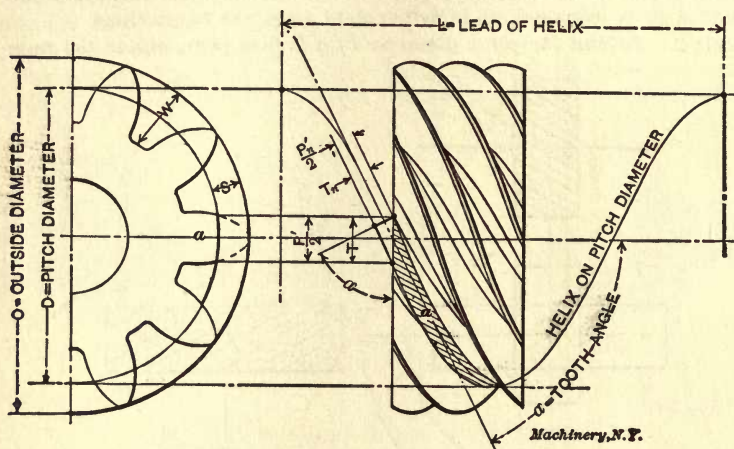


Fig. 3. Diagram of Spiral Gear, Illustrating Terms used in the Calculations

general in two different forms or classes, which may be stated as follows:

Class 1. The diametral pitch and the numbers of teeth in the two gears are given.

Class 2. A fixed center distance is given together with the velocity ratio or the numbers of teeth, with the requirement that standard cutters of even diametral pitch be used.

#### Examples of Calculations Under Class 1

Let it be required to make the necessary calculations for a pair of spiral gears in which the shafts are at right angles. Normal diametral pitch equals 3; number of teeth in gear equals 45; number of teeth in pinion equals 18.

There being no restriction in this particular case as to center distance we have to settle first on the tooth angles for the two gears. To obtain the highest efficiency, some authorities advise that the smallest tooth angle be given to the gear having the smallest number of teeth; and this angle should not, in general, run below 20 degrees. Keeping

it nearly 30 or even up to 45 would be better. On the basis  $\alpha_a = 30$ , and  $\alpha_b = 60$  degrees, we have the following calculations:

To find the pitch diameters, use Rule 2:

$$\text{Pitch diameter of gear} = \frac{45}{3 \times \cos 60^\circ} = 30 \text{ inches.}$$

$$\text{Pitch diameter of pinion} = \frac{18}{3 \times \cos 30^\circ} = 6.928 \text{ inches.}$$

To find the center distance, use Rule 3:

$$\frac{30 + 6.928}{2} = 18.464 \text{ inches.}$$

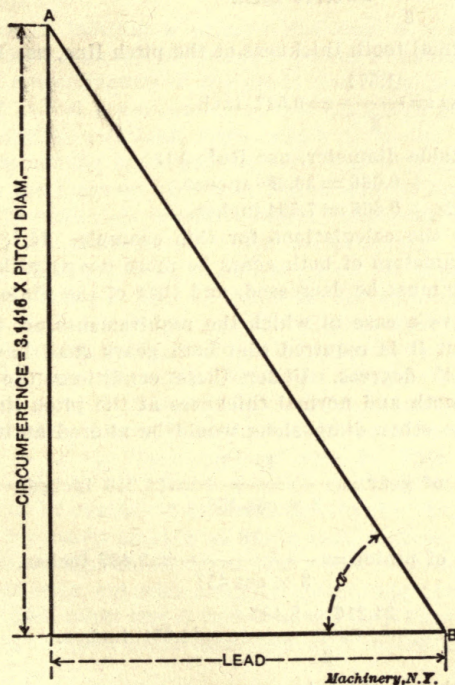


Fig. 4. Diagram Showing Relation between Pitch Diameter, Lead and Angle of Helix

To prove that the previous calculations are correct, use Rule 4:

$$45 \times \tan 60^\circ + 18 = 95.940.$$

$$2 \times 18.464 \times 3 \times \sin 60^\circ = 95.939.$$

These two results are so nearly alike that the previous calculations may be considered fully correct.

To find the number of teeth for which to select the cutter, use Rule 5:

$$\text{For gear, } \frac{45}{(\cos 60^\circ)^2} = 360.$$

$$\text{For pinion, } \frac{18}{(\cos 30^\circ)^3} = 28, \text{ approximately.}$$

To find the lead of the tooth helix, use Rule 6:

$$\text{Lead for gear} = 3.1416 \times 30 \times \cot 60^\circ = 54.38 \text{ inches.}$$

$$\text{Lead for pinion} = 3.1416 \times 6.928 \times \cot 30^\circ = 37.70 \text{ inches.}$$

To find the addendum, use Rule 7:

$$\text{Addendum} = \frac{1}{3} = 0.333 \text{ inch.}$$

To find the whole depth of tooth space, use Rule 8:

$$\text{Whole depth} = \frac{2.157}{3} = 0.719 \text{ inch.}$$

To find the normal tooth thickness at the pitch line, use Rule 9:

$$\text{Tooth thickness} = \frac{1.571}{3} = 0.527 \text{ inch.}$$

To find the outside diameter, use Rule 10:

$$\text{For gear, } 30 + 0.666 = 30.666 \text{ inches.}$$

$$\text{For pinion, } 6.928 + 0.666 = 7.594 \text{ inches.}$$

This concludes the calculations for this example. If it is required that the pitch diameters of both gears be more nearly alike, the tooth angle of the gear must be decreased, and that of the pinion increased.

Suppose we have a case in which the requirements are the same as in Example 1, but it is required that both gears shall have the same tooth angle of 45 degrees. Under these conditions the addendum, whole depth of tooth and normal thickness at the pitch line would be the same, but the other dimensions would be altered as below:

$$\text{Pitch diameter of gear} = \frac{45}{3 \times \cos 45^\circ} = 21.216 \text{ inches.}$$

$$\text{Pitch diameter of pinion} = \frac{18}{3 \times \cos 45^\circ} = 8.487 \text{ inches.}$$

$$\text{Center distance} = \frac{21.216 + 8.487}{2} = 14.851 \text{ inches.}$$

Number of teeth for which to select cutter:

$$\text{For gear, } \frac{45}{(\cos 45^\circ)^3} = 127, \text{ approximately.}$$

$$\text{For pinion, } \frac{18}{(\cos 45^\circ)^3} = 51, \text{ approximately.}$$

$$\text{Lead of helix for gear} = 3.1416 \times 21.216 \times \cot 45^\circ = 66.65 \text{ inches.}$$

$$\text{Lead of helix for pinion} = 3.1416 \times 8.487 \times \cot 45^\circ = 26.66 \text{ inches.}$$

$$\text{Outside diameter of gear} = 21.216 + 0.666 = 21.882 \text{ inches.}$$

$$\text{Outside diameter of pinion} = 8.487 + 0.666 = 9.153 \text{ inches.}$$

Examples of Calculations Under Class 2\*

In Class 2 the writer is going to make use of the term "equivalent diameter." The quotient obtained by dividing the number of teeth in a helical gear by the diametral pitch of the cutter used gives us a very useful factor for figuring out the dimensions of helical gears, so the writer has ventured to give it the name "equivalent diameter," an abbreviation of the words "diameter of equivalent spur gear," which more accurately describe it. This quantity cannot be measured on the finished gear with a rule, being only an imaginary unit of measurement.

Rule 11. *To find the equivalent diameter of a helical gear, divide the number of teeth of the gear by the diametral pitch of the cutter by which it is cut.*

The process of locating a railway line over a mountain range is divided into two parts; the preliminary survey or period of exploration, and the final determination of the grade line. The problem of designing a pair of helical gears resembles this engineering problem in having many possible solutions, from which it is the business of the designer to select the most feasible. For the exploration or preliminary survey, the diagram shown in Fig. 5 will be found a great convenience. The materials required are a ruler with a good straight edge, and a piece of accurately ruled, or, preferably, engraved, cross-section paper. If a point,  $O$ , be so located on the paper that  $BO$ , the distance to one margin line, be equal to the equivalent diameter of gear  $a$ , while  $B'O$ , the distance to the other margin line, be equal to the equivalent diameter of gear  $b$ , then (when the rule is laid diagonally across the paper in any position that cuts the margin lines and passes through point  $O$ )  $DO$  will be the pitch diameter of gear  $a$ ,  $D'O$  the pitch diameter of gear  $b$ , angle  $BO D$  the tooth angle of gear  $a$  and angle  $B'O D'$  the tooth angle of gear  $b$ . This simple diagram presents instantly to the eye all possible combinations for any given problem. It is, of course, understood that in the shape shown it can only be used for shafts making an angle of 90 degrees with each other.

The diagram as illustrated shows that a pair of helical gears having 12 and 21 teeth each, cut with a 5-pitch cutter, and having shafts at 90 degrees with each other and 5 inches apart, may have tooth angles of  $36^\circ 52'$  and  $53^\circ 8'$ , and pitch diameters of 3 inches and 7 inches, respectively.

Suppose it were required to figure out the essential data for three sets of helical gears with shafts at right angles, as follows:

- 1st. Velocity ratio 2 to 1, center distance between shafts  $2\frac{1}{4}$  inches.
- 2d. Velocity ratio 2 to 1, center distance between shafts  $3\frac{3}{8}$  inches.
- 3d. Velocity ratio 2 to 1, center distance between shafts 4 inches.

We will take the first of these to illustrate the method of procedure about to be described.

We have a center distance of  $2\frac{1}{4}$  inches and a speed ratio between driver and driven shafts of 2 to 1. The first thing to determine is

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\* MACHINERY, May, 1906.

the pitch of the cutter we wish to use. The designer selects this according to his best judgment, taking into consideration the cutters on hand and the work the gearing will have to do. Suppose he decides that 12-pitch will be about right. In Fig. 5 it will be remembered that  $DO$  was the pitch diameter of gear  $a$ , while  $D'O$  was the pitch diameter of gear  $b$ . That being the case,  $DO D'$  is equal to twice the distance between the shafts. In the problem under consideration this will be equal to  $2 \times 2\frac{1}{4}$ , or  $4\frac{1}{2}$  inches. Fig. 6 is a skeleton outline showing the operation of making the preliminary survey with rule and cross-section paper.  $AG$  and  $A'G'$  represent the margin lines of the sheet, while  $DD'$  represents the graduated straight-edge. By the

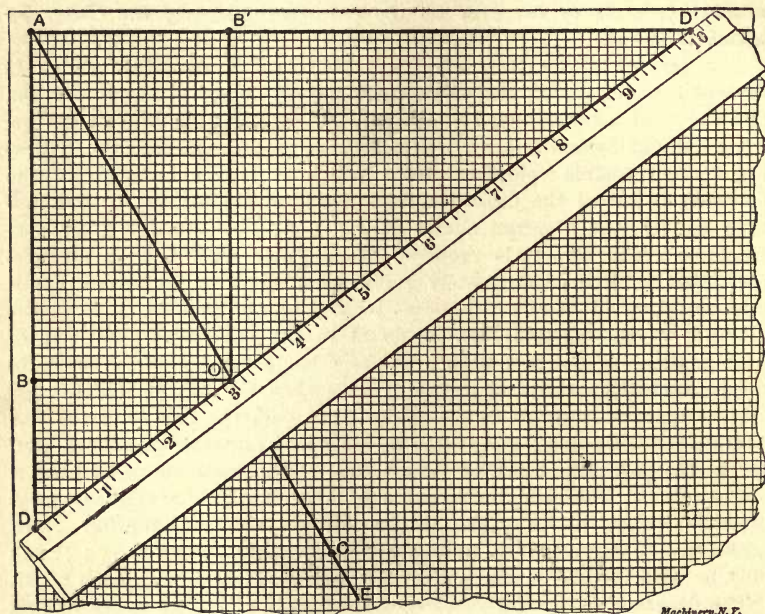


Fig. 5. Preliminary Solution with Rule and Cross-section Paper

conditions of the problem, the distance between points  $D$  and  $D'$ , where the ruler crosses the margin lines, must be equal to  $4\frac{1}{2}$  inches. There has next to be determined at what angle of inclination the ruler shall be placed in locating this line. To do this, we will first find our "ratio line." Select any point  $C$  such that  $CF'$  is to  $CF$  as 2 is to 1, which is the required ratio of our gears. Draw through point  $C$ , so located, the line  $A'E$ . Line  $A'E$  is then the ratio line, that is, a line so drawn that the measurements taken from any point on it to the margin lines will be to each other in the same ratio as the required ratio between the driving and driven gear. Now, by shifting the ruler on the margin lines, always being careful that they cut off the required distance of  $4\frac{1}{2}$  inches on the graduations, it is found that when the rule is laid as shown in position No. 1, cutting the ratio line at  $O'$ ,

the distance from the point of intersection to corner *A* is at its maximum. For the minimum value, the tooth angle is the limiting feature. For a gear of this kind, 30 degrees is, perhaps, about as small as would be advisable, so when the ruler is inclined at an angle of about 30 degrees with margin line *A G'*, and occupies position No. 2 as shown, it will cut line *A E* at *O''*, and the distance cut off from the point of intersection to corner *A* will be at its minimum value. The ruler must then be located at some intermediate position between No. 1 and No. 2.

Supposing, for example, 14 teeth in gear *a* and 28 teeth in gear *b* be tried. According to Rule 11, the equivalent diameter of gear *a* will then be  $14 \div 12$ , or 1.1666 inch; the equivalent diameter of *b* will be

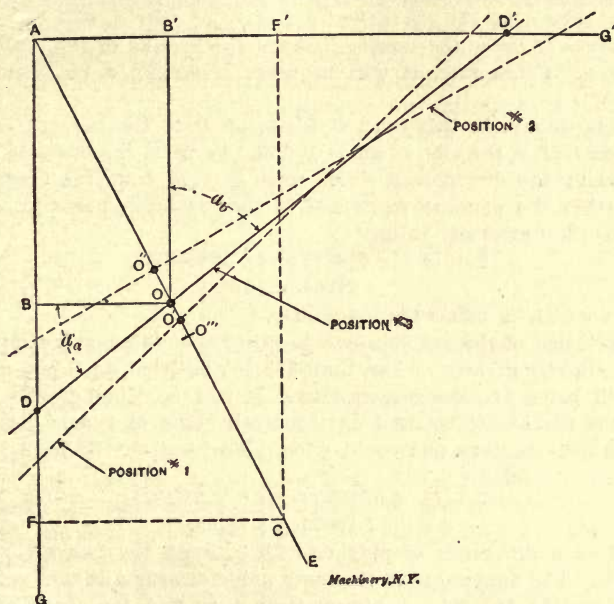


Fig. 6. Preliminary Graphical Solution for Problem No. 1

$28 \div 12$ , or 2.3333 inches. Returning to the diagram to locate the point of intersection, it will be found that point *O'''* is so located that lines drawn from it to *A G* and *A G'* will be equal to 1.1666 inch and 2.3333 inches respectively, but this is beyond point *O'*, which was found to be the outermost point possible to intersect with a  $4\frac{1}{2}$ -inch line, *D D'*. This shows that the conditions are impossible of fulfillment.

Trying next 12 teeth and 24 teeth, respectively, for the two gears, the equivalent diameters by Rule 11 will be 1 inch and 2 inches. Point *O* is now so located that *O B* equals 1 inch and *O B'* equals 2 inches. Seeing that this falls as required between *O'* and *O''*, stick a pin in at this point to rest the straight-edge against, and shift the straight-edge about until it is located in such an angular position that the

margin lines  $AG$  and  $AG'$  cut off  $4\frac{1}{2}$  inches, or twice the required distance between the shafts, on the graduations. This gives the preliminary solution to the problem. Measuring as carefully as possible,  $DO$ , the pitch diameter of gear  $a$ , is found to be about 1.265 inch diameter, and  $D'O$ , the pitch diameter of gear  $b$ , about 3.235 inches. Angle  $BOD$ , the tooth angle of gear  $a$ , measures about  $37^\circ 50'$ . Angle  $B'O D'$ , the tooth angle of gear  $b$ , would then be  $52^\circ 10'$  according to Rule 1. To determine angle  $BOD$  more accurately than is feasible by a graphical process, use the following rule:

Rule 12. *The tooth angle of gear  $a$  in a pair of mating helical gears,  $a$  and  $b$ , whose axes are  $90^\circ$  apart, must be so selected that the equivalent diameter of gear  $b$  plus the product of the tangent of the tooth angle of gear  $a$  by the equivalent diameter of gear  $a$ , will be equal to the product of twice the center distance by the sine of the tooth angle of gear  $a$ . (This rule, it will be seen, is simply a modification of Rule 4.)*

That is to say, in this case,  $O B' + (O B \times \text{the tangent of angle } BOD) = DD' \times \text{the sine of angle } BOD$ . Perform the operations indicated, using the dimensions which were derived from the diagram, to see whether the equality expressed in this equation holds true. Substituting the numerical values:

$$2 + (1 \times 0.77661) = 4.5 \times 0.61337, \\ 2.77661 = 2.76016,$$

a result which is evidently inaccurate.

The solution of the problem now requires that other values for angle  $BOD$ , slightly greater or less than  $37^\circ 50'$ , be tried until one is found that will bring the desired equality. It will be found finally that if the value of  $38^\circ 20'$  be used as the tooth angle of gear  $a$ , the angle is as nearly right as one could wish. Working out Rule 12 for this value:

$$2 + (1 \times 0.79070) = 4.5 \times 0.62024, \\ 2.79070 = 2.79108.$$

This gives a difference of only 0.00038 between the two sides of the equation. The final value of the tooth angle of gear  $a$  is thus settled as being equal to  $38^\circ 20'$ . Applying Rule 1 to find the tooth angle of gear  $b$  we have:  $90^\circ - 38^\circ 20' = 51^\circ 40'$ . The next rule relates to finding the pitch diameter of the gears.

Rule 13. *The pitch diameter of a helical gear equals the equivalent diameter divided by the cosine of the tooth angle; (or the equivalent diameter multiplied by the secant of the tooth angle).* This rule is a modification of Rule 2.

If a table of secants is at hand, it will be somewhat easier to use the second method suggested by the rule, since multiplying is usually easier than dividing. Using in this case, however, the table of cosines, and performing the operation indicated by Rule 13, we have for the pitch diameter of gear  $a$ :

$$1 \div 0.78442 = 1.2748, \text{ or } 1.275 \text{ inch, nearly;}$$

and for the pitch diameter of gear  $b$ :

$$2 \div 0.62024 = 3.2245, \text{ or } 3.225 \text{ inches, nearly.}$$

To check up the calculations thus far, the pitch diameter of the two gears thus found may be added together. The sum should equal twice the center distance, thus:

$$1.275 + 3.225 = 4.500,$$

which proves the calculations for the angle.

Applying Rule 10 to gear *a*:

$$1.2748 + (2 \div 12) = 1.2748 + 0.1666 = 1.4414 = 1.441 \text{ inch, nearly.}$$

For gear *b*:

$$3.2245 + (2 \div 12) = 3.2245 + 0.1666 = 3.3911 = 3.391 \text{ inches, nearly.}$$

In cutting spur gears of any given pitch, different shapes of cutters are used, depending upon the number of teeth in the gear to be cut. For instance, according to the Brown & Sharpe system for involute gears, eight different shapes are used for a gear from 12 teeth to a rack. The fact that a certain cutter is suited for cutting a 12-tooth spur gear is no sign that it is suitable for cutting a 12-tooth helical gear, since the fact that the teeth are cut on an angle alters their shape considerably. To find out the number of teeth for which the cutter should be selected, use Rule 5.

Applying Rule 5 to gear *a*:

$$12 \div 0.784^{\circ} = 12 \div 0.4818 = 25 \text{ —.}$$

and for gear *b*:

$$24 \div 0.620^{\circ} = 24 \div 0.2383 = 100 \text{ +,}$$

giving, according to the Brown & Sharpe catalogue, cutter No. 5 for gear *a* and cutter No. 2 for gear *b*.

In gearing up the head of the milling machine to cut these gears it is necessary to know the lead of the helix or "spiral" required to give the tooth the proper angle. To find this, use Rule 6. In solving problems by this rule, as for Rule 5, it will be sufficient to use trigonometrical functions to three significant places only, this being accurate enough for all practical purposes. Solving by Rule 6 to find the lead for which to set up the gearing in cutting *a*:

$$1.275 \times 1.265 \times 3.14 = 5.065, \text{ or } 5 \frac{1}{16} \text{ inches, nearly;}$$

for gear *b*:

$$3.225 \times 0.791 \times 3.14 = 8.010, \text{ or } 8 \frac{3}{32} \text{ inches, nearly.}$$

The lead of the helix must be, in general, the adjustable quantity in any spiral gear calculation. If special cutters are to be made, the lead of the helix may be determined arbitrarily from those given in the milling machine table; this will, however, probably necessitate a cutter of fractional pitch. On the other hand, by using stock cutters and varying the center distance slightly, we might find a combination which would give us for one gear a lead found in the milling machine table, but it would only be chance that would make the lead for the helix in the mating gear also of standard length. It is then generally better to calculate the milling machine change gears according to the usual methods to suit odd leads, rather than to adapt the other conditions to suit an even lead. It will be found in practice that the

lead of the helix may be varied somewhat from that calculated without seriously affecting the efficiency of the gears.

The remaining calculations relating to the proportions of the teeth do not vary from those for spur gears and are here set down for the sake of completeness only.

The addendum of a standard gear is found by Rule 7:

For gears  $a$  and  $b$  this will give:

$$1 \div 12 = 0.0833 \text{ inch.}$$

The whole depth of the tooth is found by Rule 8:

This gives for gears  $a$  and  $b$ :

$$2.157 \div 12 = 0.1797 \text{ inch.}$$

The thickness of the tooth is found by Rule 9:

For gears  $a$  and  $b$  of our problem this gives:

$$1.571 \div 12 = 0.1309 \text{ inch.}$$

This completes all the calculations required to give the essential data for making our first pair of helical gears. To illustrate the variety of conditions for which these problems may be solved, the other cases will be worked out somewhat differently. In the case just considered no allowance was made for possible conditions which might have limited the dimensions of the gears, and the problem was solved for what might be considered good general practice. Gear  $a$ , however, might have been too small to put on the shaft on which it was intended to go, while gear  $b$  might have been too large to enter the space available for it. If, as we may assume, these gears are intended to drive the camshaft of a gas engine, the solution would probably be unsatisfactory. Case No. 2 will therefore be solved for a center distance of  $3\frac{3}{4}$  inches as required, but the two gears will be made of about equal diameter. Fig. 7 shows the preliminary graphical solution of this problem, the reference letters in all cases being the same as in Fig. 6. With a 10-pitch cutter, if this suited the judgment of the designer, 15 teeth in gear  $a$  and 30 teeth in gear  $b$  would require that the point of intersection on the ratio line  $A E$  be located at  $O$  where  $B O$  equals the equivalent diameter of gear  $a$ , which equals  $1\frac{1}{2}$  inch, while  $B' O$  equals the equivalent diameter of gear  $b$ , or 3 inches, both calculated in accordance with Rule 11. The required condition now is that  $D O$  be approximated to  $D' O$ ; that is to say, that the pitch diameters of the two gears be about equal. After continued trial it will be found impossible to locate  $O$ , using a cutter of standard diametral pitch, so that  $D O$  and  $D' O$  shall be equal, and at the same time have  $D D'$  equal to twice the required center distance, which is  $2 \times 3\frac{3}{4}$  inches or  $6\frac{3}{4}$  inches. If this center distance could be varied slightly without harm,  $B D$  could be taken as equal to  $A B$ ; then it would be found that a line drawn from  $D$  through  $O$  to  $D'$ , though giving a somewhat shortened center distance, would make two gears of exactly the same pitch diameter.

Drawing line  $D O D'$ , however, as first described to suit the conditions of the problem, and measuring it for a preliminary solution the following results are obtained: The tooth angle of gear  $a$  = angle

$BOD = 63^\circ 45'$ ; and the tooth angle of gear  $B = \text{angle } B'OD' = 90^\circ - 63^\circ 45' = 26^\circ 15'$ , according to Rule 1. Performing the operations indicated in Rule 12 to correct these angles, it is found that when the tooth angle of gear  $a$  is  $63^\circ 54'$ , and that for gear  $b$  is  $26^\circ 6'$ , the equation of Rule 12 becomes:

$$3 + (15 \times 2.04125) = 6.75 \times 0.89803$$

$$6.06187 = 6.06170$$

which is near enough for all practical purposes. The other dimensions

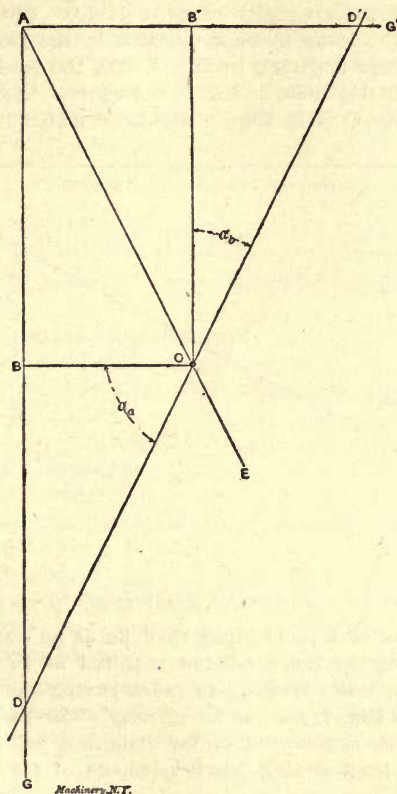


Fig. 7. Solution of Problem No. 2 for Equal Diameters

are easily obtained as before by using the remaining rules.

To still further illustrate the flexibility of the helical gear problem, the third case, for a center distance of 4 inches, will be solved in a third way. It is shown in MacCord's "Kinematics" that to give the least amount of sliding friction between the teeth of a pair of mating helical gears, the angles should be so proportioned that, in our diagrams, line  $DD'$  will be approximately at right angles to ratio line  $AE$ . On the other hand, to give the least end thrust against the bearings, line  $DD'$  should make an angle of  $45^\circ$  with the margin lines  $AG$  and  $A'G'$ , in

the case of gears with axes at an angle of  $90^\circ$ , as are the ones being considered. The first example worked out in detail was solved in accordance with "good practice," and line  $DD'$  was located about one-half way between the two positions just described, thus giving in some measure the advantage of a comparative absence of sliding friction, combined with as small degree of end thrust as is practicable. To illustrate some of the peculiarities of the problem, Case 3 will now be solved to give the minimum amount of sliding friction, neglecting entirely the end thrust, which is considered to be taken up by ball thrust bearings or some equally efficient device. On trial it will be found that, with the same number of teeth in the gear and with the same pitch as in Case 2, giving in Fig. 8,  $BO$ , the equivalent diameter of gear  $a$ , a value of  $1\frac{1}{2}$  inch, and  $B'O$ , the equivalent diameter of gear  $b$ , a value of 3 inches, as in Fig. 7, line  $DD'$  which is equal to twice

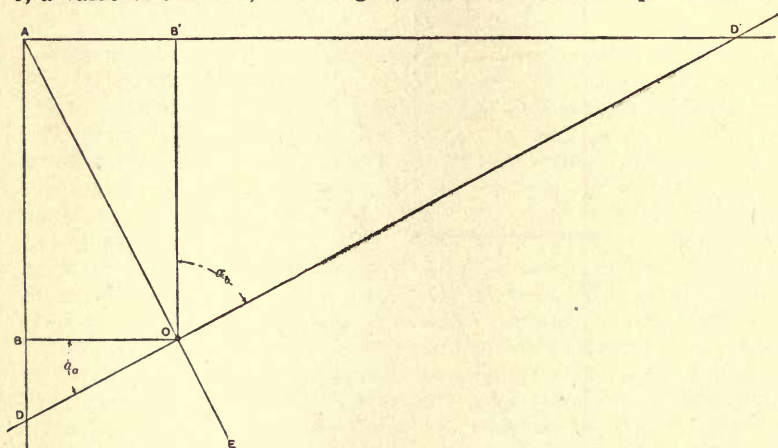


Fig. 8. Solution of Problem No. 3 for Minimum Sliding Friction

the center distance, or 8 inches, can then lie at an angle of about  $90^\circ$  with  $AE$ , thus meeting the condition required as to sliding friction. Thus this diagram, while relating to gears having the same pitch and number of teeth as Fig. 7, yet has an entirely different appearance, and gives different tooth angles and center distances, solving the problem as it does for the least sliding friction instead of for equal diameters of gears.

Measuring the diagram as accurately as may be, the following results are obtained: Tooth angle of gear  $a = BOD = 28^\circ$ ; tooth angle of gear  $b = \text{angle } B'OD' = 90^\circ - 28^\circ = 62^\circ$ . This is the preliminary solution. After accurately working it out by the process before described, we have as a final solution, tooth angle of gear  $a = 28^\circ 28'$ ; tooth angle of gear  $b = 61^\circ 32'$ . From this the remaining data can be calculated.

For designers who feel themselves skillful enough to solve such problems as these graphically without reference to calculations, the diagram may be used for the final solution. The variation between the results

obtained graphically and those obtained in the more accurate mathematical solution is a measure of the skill of the draftsman as a graphical mathematician. The method is simple enough to be readily copied in a note book or carried in the head. If the graphical method is to be used entirely, it will be best not to trust to the cross-section paper, which may not be accurately ruled; instead skeleton diagrams like those shown in Figs. 6, 7 and 8 may be drawn. For rough solutions, however, to be afterward mathematically corrected, as in the examples considered in this chapter, good cross-section paper is accurate enough. It permits of solving a problem without drawing a line. Point *O* may be located by reading the graduations; a pin inserted here may be used as a stop for the rule, from which the diameter and center distance are read directly; dividing  $AD$ , read from the paper, by  $DD'$ , read from the rule, will give the sine of the tooth angle of the gear  $a$ .

#### Formulas for Spiral Gearing

For sensible people, who prefer their rules to be embodied in formulas, the appended list has been prepared, using the following reference letters, which agree in general with the nomenclature of the Brown & Sharpe gear books.

- $N_a$  = number of teeth in gear  $a$ ,
- $N_b$  = number of teeth in gear  $b$ ,
- $P_n$  = normal diametral pitch or pitch of cutter,
- $\gamma$  = center angle,
- $\alpha$  = angle of tooth with axis,
- $D$  = pitch diameter,
- $C$  = center distance,
- $N'$  = number of teeth for which to select cutter,
- $L$  = lead of tooth helix,
- $S$  = addendum,
- $W$  = whole depth of tooth,
- $T_n$  = normal thickness of tooth at pitch line,
- $O$  = outside diameter.

Where subscript letters  $a$  and  $b$  are used, reference is made to gears  $a$  and  $b$ , as for instance, " $N_a$ " and " $N_b$ ," where the letter  $N$  refers to the number of teeth in gears  $a$  and  $b$ , respectively, of a pair of gears  $a$  and  $b$ .

$$\gamma = \alpha_a + \alpha_b \quad (1)$$

$$D = \frac{N}{P_n \cos \alpha} \quad (2)$$

$$C = \frac{D_a + D_b}{2} \quad (3)$$

$$N_b + (N_a \times \tan \alpha_a) = 2 C P_n \times \sin \alpha_a \quad (4)$$

$$N' = \frac{N}{(\cos \alpha)^3} \quad (5)$$

$$L = \pi D \times \cot \alpha \quad (6)$$

$$S = \frac{1}{P_n} \quad (7)$$

$$W = \frac{2.157}{P_n} \quad (8)$$

$$T_n = \frac{1.571}{P_n} \quad (9)$$

$$O = D + 2S \quad (10)$$

#### Examples of Spiral Gear Problems\*

A number of examples will be given in the following, which can be solved by simple modifications of the methods outlined for problems of Class 2. The same reference letters are used as before.

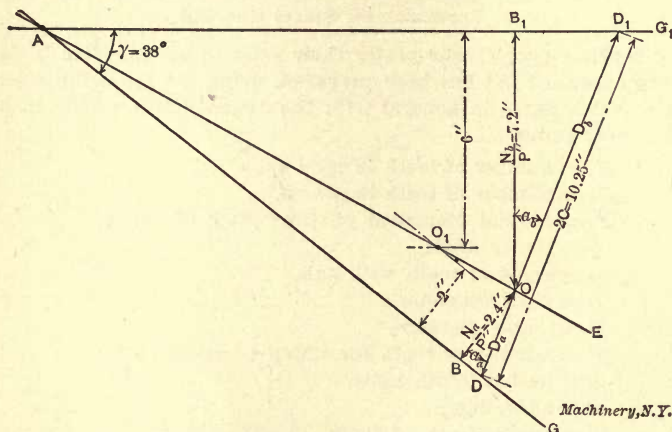


Fig. 9

**Example 1.**—Find the essential dimensions for a pair of spiral gears, velocity ratio 3 to 1, center distance between shafts  $5\frac{1}{8}$  inches, angle between shafts 38 degrees.

First obtain a preliminary solution by the diagram shown in Fig. 9. Draw lines  $AG$  and  $AG_1$  making an angle  $\gamma$  with each other equal to 38 degrees, the angle between the axes. Locate the ratio line  $AE$  by finding any point such as  $O_1$  between  $AG$  and  $AG_1$ , that is distant from each of them in the same ratio as that desired for the gearing. In the case shown, it is 6 inches from  $AG_1$ , and 2 inches from  $AG$ , which is in the ratio of 3 to 1 as required. Through  $O_1$  draw line  $AE$  which may be called the ratio line. Select a trial number of teeth and pitch of cutter for the two gears, such, for instance, as 36 teeth for the gear and 12 for the pinion, and with 5 diametral pitch of the cutter. The diameter of a spur gear of the same pitch and number of teeth as the gear would be  $36 \div 5 = 7.2$  inches. Find the point  $O$

\* MACHINERY, December, 1908.

on  $AE$ , which is 7.2 inches from  $AG_1$ . This point will be 2.4 inches from  $AG$ , if  $AE$  is drawn correctly.

Now apply a scale to the diagram, with the edge passing through  $O$  and with the zero mark on line  $AG$ , shifting it to different positions until one is found in which the distance across from one line to another ( $DD_1$  in the figure) is equal to twice the center distance, or 10.25 inches. If a position of the rule cannot be found which will give this distance between lines  $AG$  and  $AG_1$ , new assumptions as to number of teeth and diametral pitch of the gear and pinion must be made, which will bring point  $O$  in a location where line  $DD$ , may be properly laid out.  $DD_1$  being drawn, the problem is solved graphically. The tooth angle of the gear is  $B_1OD_1$ , or  $a_b$ , while that of the pinion

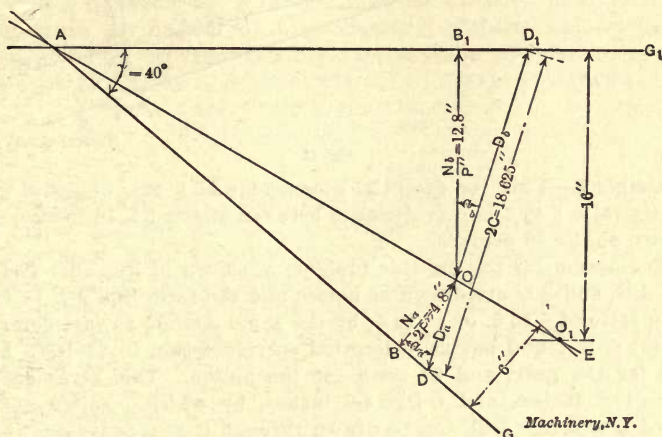


Fig. 10

is  $BOD$ , or  $a_a$ .  $OD_1$  will be the pitch diameter of the gear, and  $OD$  the pitch diameter of the pinion.

To obtain the dimensions more accurately than can be done by the graphical process, the pitch diameters should be figured from the tooth angles we have just found. To do this, divide the dimensions  $OB_1$  and  $OB$  for gear and pinion, by the cosine of the tooth angles found for them. If they measure on the diagram, for instance, 21 degrees 50 minutes and 16 degrees 10 minutes respectively (note that the sum of  $\alpha_a$  and  $\alpha_b$  must equal  $\gamma$ ), the calculation will be as follows:

$$\begin{array}{r} 7.2 \div 0.92827 = 7.7563 = D_b \\ 2.4 \div 0.96046 = 2.4988 = D_a \\ \hline 10.2551 = 2C \end{array}$$

The value we thus get, 10.2551 inches, for twice the center distance, is somewhat larger than the required value, 10.250 inches. We have now to assume other values for  $\alpha_a$  and  $\alpha_b$ , until we find those which give pitch diameters whose sum equals twice the center distance. Assume, for instance, that  $\alpha_b = 21$  degrees 43 minutes, then  $\alpha_a = 33$  de-

grees — 21 degrees 43 minutes = 16 degrees 17 minutes. We now have:

$$7.2 \div 0.92902 = 7.7501 = D_b$$

$$2.4 \div 0.95989 = 2.5003 = D_a$$

$$10.2504 = 2 C$$

This value for twice the center distance is so near that required that we may consider the problem as solved. The other dimensions for the outside diameter, lead, etc., may be obtained as for spiral gears at right angles, and as described in the previous part of this chapter.

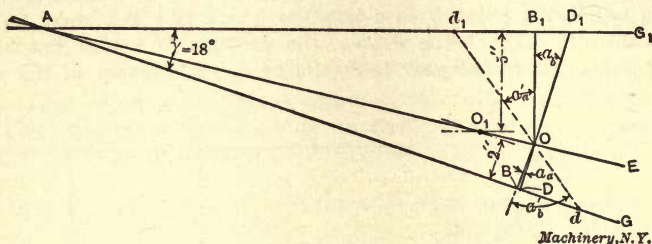


Fig. 11

*Example 2.*—Find the essential dimensions of a pair of spiral gears, velocity ratio 8 to 3, center distance between shafts  $9 \frac{5}{16}$  inches, angle between shafts 40 degrees.

The diagram for solving this problem is shown in Fig. 10. The axis lines  $A G_1$  and  $A G$  are drawn as before and the ratio line  $A E$  is drawn in the ratio of 8 to 3, or 16 to 6, by the same method as just described. A point  $O$  is found having a location corresponding to 64 teeth and 5 pitch for the gear, and 24 teeth for the pinion. This gives distance  $O B_1 = 12.8$  inches, and  $O B = 4.8$  inches, by which position  $O$  is so located that a line  $D D_1$  can be drawn through it at a convenient angle,

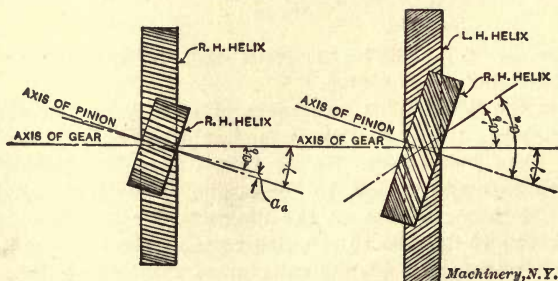


Fig. 12

Fig. 13

and with a length equal to twice the center distance, or 18.625 inches. We measure the angle for a preliminary graphical solution as before, and then by trial find the final solution as follows, in which angle  $\alpha_a$  is 17 degrees 45 minutes, and  $\alpha_a$  is 22 degrees 15 minutes'

$$12.8 \div 0.95240 = 13.4397 = D_b$$

$$4.8 \div 0.92554 = 5.1862 = D_a$$

$$18.6259 = 2 C$$

This gives the value of twice the center distance near enough for gears of this size.

*Example 3.*—Find the essential dimensions for a pair of spiral gears, velocity ratio 5 to 2, center distance between shafts  $4\frac{1}{16}$  inches, angle of shafts 18 degrees.

The diagram for solving this problem is shown in Fig. 11. The axis lines  $AG_1$  and  $AG$  are drawn as before, and the ratio line  $AE$  is drawn in the ratio of 5 to 2, by the same method as just described. A point  $O$  is found having a location corresponding to 45 teeth and 8 pitch for the gear, and 18 teeth for the pinion. This gives distance  $OB_1 = 5.625$  inches, and  $OB = 2.250$  inches, in which position  $O$  is so located that line  $DD_1$  can be drawn through it at a convenient angle, and with a length equal to twice the center distance, or 8.125 inches. We measure the angles for a preliminary mathematical solution as before, and then by trial find the final solution as follows, in which angle  $\alpha_b$  is 16 degrees 45 minutes and  $\alpha_a$  is 1 degree 15 minutes:

$$\begin{aligned} 5.625 \div 0.95757 &= 5.8742 = D_b \\ 2.250 \div 0.99976 &= 2.2505 = D_a \end{aligned}$$

$$8.1247 = 2C$$

It is often a matter of great difficulty, when the center angle  $\gamma$  is as small as in this case, to find a location for point  $O$  such that standard cutters can be used, and that line  $DD_1$  can be drawn of the proper length through  $O$  without bringing  $D$  to the left of  $B$ , or  $D_1$  to the left of  $B_1$ . It will be noticed in this case that to make the center distance come right, angle  $\alpha_a$  had to be made very small, so that the pinion is practically a spur gear. In some cases, to get the proper center distance, it may be necessary to so draw line  $DD_1$  that one of the tooth angles is measured on the left side of  $BO$  or  $B_1O$ . Such a case, for instance, is shown in the position of  $d_1Od$ . When a line has to be drawn like this, the tooth angles  $\alpha'_a$  and  $\alpha'_b$  are opposite in inclination, instead of having them, as usual, either both right hand or both left hand. In Fig. 12 are shown gears drawn in accordance with the location of line  $DD_1$  of Fig. 11, while Fig. 13 shows a pair drawn in accordance with  $dd_1$  of the same diagram, which will illustrate the state of affairs met with in cases of this kind. This expedient of making one spiral gear right-hand and one left-hand should never be resorted to except in case of extreme necessity, as the construction involves a very wasteful amount of friction from the sliding of the teeth on each other as the gears revolve.

#### Demonstration of Grant's Formula

As already mentioned, the number of teeth for which the cutter should be selected for cutting a helical gear, is found to be the formula

$$N' = \frac{N}{\cos^2 \alpha}$$

in which  $N'$  = number of teeth for which cutter is selected,

$N$  = actual number of teeth in helical gear,

$\alpha$  = angle of tooth with axis.

Note that  $\cos^2 \alpha$  is equivalent to  $(\cos \alpha)^2$ .

A demonstration of this formula was presented by Mr. H. W. Henes in *MACHINERY*, April, 1908. This demonstration is as follows:

Let  $P_n$  be the perpendicular distance between two consecutive teeth on the spiral gear, and let  $D_1$  be the diameter of the spiral gear. Let the gear be represented as in Fig. 14, and pass a plane through it perpendicular to the direction of the teeth. The section will be an ellipse as shown in  $CEDF$ . Designate the semi-major and semi-minor axes by  $a$  and  $b$  respectively.

Now  $N'$  is the number of teeth which a spur gear would have if its

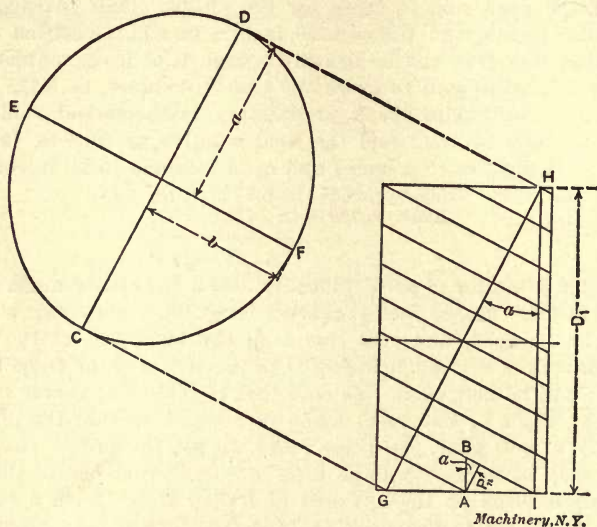


Fig. 14. Diagram for Deriving the Formula for Determining Spur Gear Cutter for Cutting Spiral Gears

radius were equal to the radius of curvature of the ellipse at  $E$ . Therefore, it is required to determine the radius of this curvature of the ellipse. This is done as follows:

From the figure we have:

$$2b = \text{axis } EF = D_1 \quad (11)$$

$$2a = \text{axis } CD = GH = \frac{HI}{\cos \alpha} = \frac{D_1}{\cos \alpha} \quad (12)$$

From (11) and (12) we have for  $a$  and  $b$ ,

$$b = \frac{D_1}{2} \quad (13)$$

$$a = \frac{D_1}{2 \cos \alpha} \quad (14)$$

It is known, and shown by the methods of calculus, that the minimum curvature of an ellipse, that is, the curvature at  $E$  or  $F$ , equals

$\frac{b}{a^2}$ . Taking the values of  $a$  and  $b$  found in (13) and (14), we have the curvature at  $E$ :

$$\text{Curvature} = \frac{b}{a^2} = \frac{\frac{D_1}{2}}{\frac{D_1^2}{4 \cos^2 a}} = \frac{4 D_1 \cos^2 a}{2 D_1^2} = \frac{2 \cos^2 a}{D_1} \quad (15)$$

It is also shown in calculus that the curvature is equal to  $\frac{1}{R}$ , where  $R$  is the radius of curvature at the point  $E$ . Therefore from (15) we have:

$$\frac{1}{R} = \frac{2 \cos^2 a}{D_1}, \text{ whence } R = \frac{D_1}{2 \cos^2 a} \quad (16)$$

Formula (16) can also be arrived at directly, without reference to the minimum curvature of the ellipse, by introducing the formula for the radius of curvature in the first place. The curvature is simply the reciprocal value of the radius of curvature, and is only a comparative means of measurement. The radius of curvature of an ellipse at the end of its short axis is  $\frac{a^2}{b}$ , from which formula (16) may be derived directly by introducing the values of  $a$  and  $b$  from equations (13) and (14).

Having now found the radius of curvature of the ellipse at  $E$ , we proceed to find the number of teeth which a spur gear of that radius would have. From Fig. 14 we have:

$$AB = \frac{P_n}{\cos a} \quad (17)$$

Now, if  $AB$  be multiplied by the number of teeth of the spiral gear, we shall obtain a quantity equal to the circumference of the gear; that is:

$$AB \times N = \pi D_1, \text{ and since } AB = \frac{P_n}{\cos a} \text{ from (17)}$$

$$\frac{P_n}{\cos a} \times N = \pi D_1 \quad (18)$$

Since  $N'$  is the number of teeth which a spur gear of radius  $R$  would have, then,

$$N' = \frac{2 \pi R}{P_n} \quad (19)$$

In equation (19) the numerator of the fraction is the circumference of the spur gear whose radius is  $R$ , and the denominator is the circular pitch corresponding to the cutter.

From equation (16) we have:

$$R = \frac{D_1}{2 \cos^2 \alpha}$$

Substituting this value of  $R$  in (19), we have:

$$N' = \frac{2 \pi D_1}{P_n \times 2 \cos^2 \alpha} \quad (20)$$

From equation (18) we have:

$$D_1 = \frac{N P_n}{\pi \cos \alpha} \quad (21)$$

Substitute this value of  $D_1$  in equation (20) and we have:

$$N' = \frac{2 \pi N P_n}{2 P_n \pi \cos^3 \alpha}$$

or

$$N' = \frac{N}{\cos^3 \alpha} \quad (22)$$

Since  $N$  is the number of teeth in our spiral gear and  $N'$  is the number of teeth in a spur gear which has the same radius as the radius of curvature of the helix above referred to, this is the equivalent of saying that the cutter to be used should be correct for a number of teeth which can be obtained by dividing the actual number of teeth in the gear by the cube of the cosine of the tooth angle. Since the cosine of angle is always less than unity, its cube will be still less, so  $N'$  is certain to be greater than  $N$ , which will account for the fact that spiral gears of less than 12 teeth can be cut with the standard cutters.

## CHAPTER II

### DIAGRAMS FOR DESIGNING SPIRAL GEARS\*

Great difficulties are usually experienced in designing spiral gears, and these difficulties are greatly accentuated when one has to design them for two shafts whose center distance cannot be altered to suit the gears, and also when the angle between the shafts is not a right angle, and the speed ratio is not equal. The general practice is to work out the gears by lengthy mathematics, and should the answer not come out as desired, then a new trial is made, varying either one or the other factor, until the angles and diameters are correct. This method of "cut and try" entails a great deal of work and waste of time. The following method, together with the diagrams used with it, will remove some of the difficulties, and enable one to arrive at the data required in a very short time. The method adopted is graphical, but the results may be checked by simple figuring.

As the pitch diameter, spiral angle, and circular pitch are interdependent, they cannot be considered as a starting point in solving the problem, because they are not known. The starting point, therefore, must be the speed ratio, and some idea of the strength required, together with the center distance. These factors, as a rule, can easily be ascertained. As it is common usage to employ ordinary spur gear cutters of regular diametral pitches for cutting spiral gears, the normal pitch, or distance from one tooth to the next measured at right angles to the tooth, must be the same as the pitch of a spur gear for which the cutter to be used is intended; therefore the corresponding diametral pitch and the speed ratio must be the initial data, all others being obtained afterwards.

Three diagrams are given for the graphical solution of spiral gears. The diagram in Fig. 15 shows the relation between the quotient of number of teeth  $\div$  diametral pitch, spiral angles, and pitch diameters.

The quotient 
$$\frac{\text{number of teeth}}{\text{diametral pitch}}$$
 is commonly termed "equivalent diameter," and will be so referred to in the following.

The diagram in Fig. 16 shows the relation between the diametral pitch, the number of teeth, and the equivalent diameter. Finally, the diagram in Fig. 17 shows the relation between the pitch diameter, the spiral angle, and the lead of the helix. We will now proceed to give some typical examples illustrating the use of the diagrams.

*Example 1.* Given a gear having 24 teeth, 6 diametral pitch, and a spiral angle of 40 degrees. Find the pitch diameter.

First obtain the value of the ratio, number of teeth  $\div$  diametral pitch, which, in this case, can be obtained without referring to dia-

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\* MACHINERY, October, 1908.

gram Fig. 16, being simply  $24 \div 6 = 4$ . Locate 4 on the horizontal line in diagram Fig. 15, and project vertically until the line from figure 4 intersects the line for 40 degrees spiral angle. Then follow the circular arc from this point, either to the right or downward, reading off 5.22 on the corresponding scale, this being the pitch diameter. Should the diameter be required accurately, we can figure it by the formula:

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{\text{Diametral pitch}} \times \frac{1}{\cos \text{spiral angle}}$$

$$= 4 \times \frac{1}{\cos 40 \text{ deg.}} = 5.222 \text{ inches.}$$

This also gives a check of the result obtained by means of the dia-

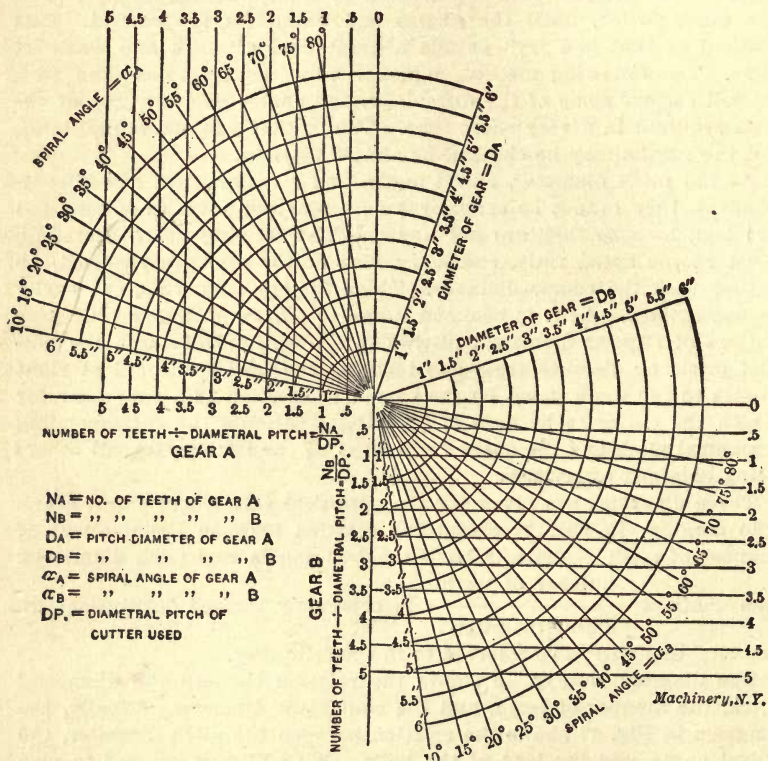


Fig. 15. Diagram of Relation between Number of Teeth, Diametral Pitch, Spiral Angles and Pitch Diameters

gram. The lead of the helix is now obtained from Fig. 17, by projecting the pitch diameter 5.22 horizontally to the radial line for the spiral angle, and then, following the vertical line to the lead scale at the bottom of the diagram, we find, in this case, a lead of 19.6 inches. Of course, the outside diameter of the blank would be  $5.222 +$

$2 \times 1/6 = 5.555$  inches, which is the pitch diameter  $\div$  2 times the addendum.

*Example 2.* Required two gears which are to be equal in all respects, the diametral pitch being 8, and the centers to be approximately 4 inches apart.

As the centers are not fixed, the gears in this case may be made with 45 degrees spiral angle, and the center distance may be slightly adjusted to suit the pitch diameters. Referring to Fig. 15, follow the circular arc from diameter of gear = 4 inches, until it intersects the radial line for 45 degrees spiral angle; then follow the vertical line down to the scale of the ratio between the number of teeth and diam-

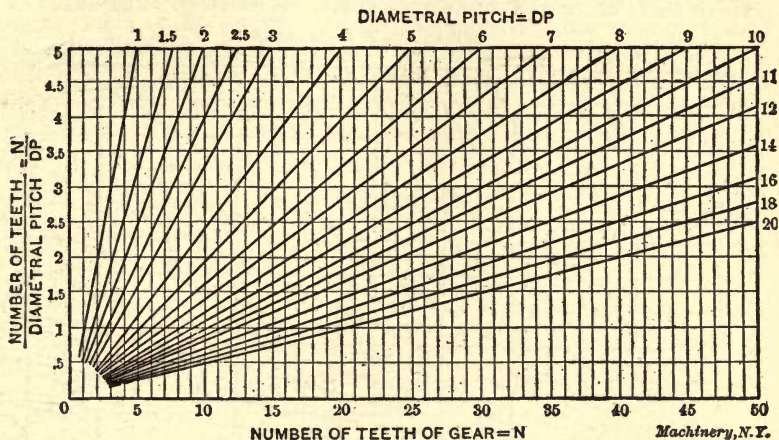


Fig. 16. Relation between Diametral Pitch, Number of Teeth, and Quotient of Number of Teeth divided by Diametral Pitch

etral pitch, which is found to be 2.82. Then, from Fig. 16, we find that with this ratio and 8 diametral pitch, the number of teeth is not a whole number, but the nearest number is 23, giving a ratio of 2.875 instead of 2.82, which, by reversing the process and referring to diagram Fig. 15, gives a pitch diameter of 4.07 inches. These results may be checked as follows:

$$\begin{aligned} \text{Pitch diameter} &= \frac{\text{No. of teeth}}{\text{Diametral pitch}} \times \frac{1}{\cos 45 \text{ deg.}} \\ &= 2.875 \times \frac{1}{0.707} = 4.07 \text{ inches.} \end{aligned}$$

The outside diameter is  $4.07 + 2 \times 0.125 = 4.32$ . The lead, as obtained from diagram Fig. 17, in the same way as in Example 1, is 12.79 inches.

*Example 3.* Required a pair of spiral gears having a normal pitch corresponding to 10 diametral pitch, having a given center distance of  $2\frac{1}{2}$  inches approximately, the sum of the spiral angles being 90 degrees, and the speed ratio equal to 5 to 1.

In this case both portions of diagram Fig. 15 are used, the upper part

being employed for one gear and the lower part for the other, the easiest way being to get a strip of paper with two lines marked on its edge 5 inches (twice the center distance) apart, drawn to the same scale as the diagram. Move this strip of paper on the diagram (so that the edge of the strip passes through the center), as indicated at A, Fig. 18, until the lines marked coincide with the points where the ratio of the equivalent diameters equals 5 to 1, and then determine from Fig. 16 that these diameters also give whole numbers of teeth with 10 diametral pitch. We find that 0.5 and 2.5 at 78 degrees and 12 degrees are two such positions, and also 0.6 and 3.0 at 70 degrees and 20 degrees. If we use the latter values, we will have 6 teeth and 30 teeth at 70 and 20 degrees angle, respectively. The exact di-

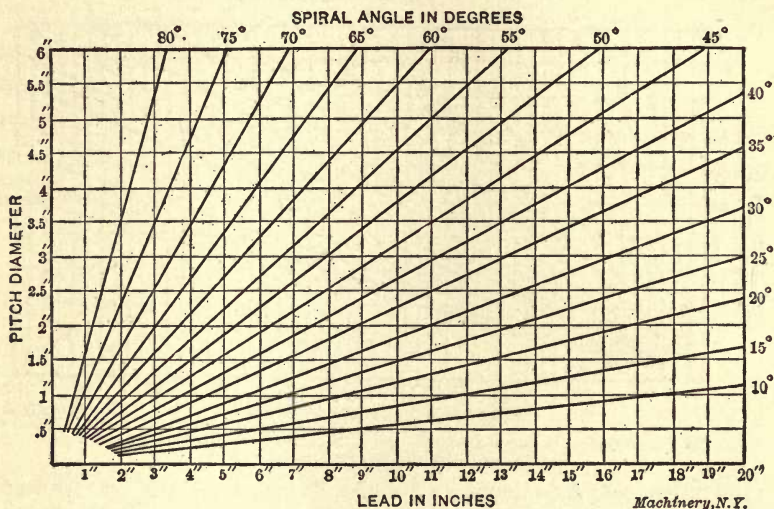


Fig. 17. Relation between Pitch Diameter, Spiral Angle and Lead of Helix

ameters can now be determined, as in our previous problem, and are 1.75 and 3.19 inches, respectively, the outside diameters being 0.2 inch larger, or 1.95 and 3.39 inches, respectively. This gives the center distances 2.47. These values can now be obtained from the formulas as before.

*Example 4.* Required a pair of spiral gears, having a fixed center distance of 4.5 inches, running at equal speeds, the diametral pitch being 7. The method of procedure is similar to that of the last example, using a strip of paper having a distance of 9 inches marked on the edge in the proper scale, as indicated at B in Fig. 18. At about 40 degrees spiral angle we find in Fig. 15 the ratio of number of teeth to diametral pitch to equal 3.14. This ratio must be adjusted on diagram Fig. 16, as previously shown, so as to enable one to get a whole number of teeth with 7 diametral pitch, this number being in this case 22. The ratio is then 3.143, and following from this in Fig. 15 to the 40-degree line, one obtains a pitch diameter of about 4.1 inches for one gear, and at 50 degrees about 4.9 inches for the other.

The spiral angles should now be carefully checked mathematically as follows:

$$\cos \text{ spiral angle (first gear)} = 3.143 \times \frac{1}{4.1} = 0.766; \text{ spiral angle} = 40 \text{ deg.}$$

$$\cos \text{ spiral angle (second gear)} = 3.143 \times \frac{1}{4.9} = 0.642;$$

spiral angle = 50 deg., nearly.

Now obtain the leads from diagram Fig. 17 in the same way as before, giving the leads of the gears 15.4 and 12.9 inches, respectively.

*Example 5.* Required a pair of spiral gears, the axes of which are at an angle of 120 degrees; center distance 4.125; the ratio of equivalent diameters should be as 2 to 3, and the diametral pitch equals 5.

We require first of all two numbers representing the equivalent diameters, these two numbers bearing the ratio to each other of 2 to 3, and giving a whole number of teeth with 5 diametral pitch. These

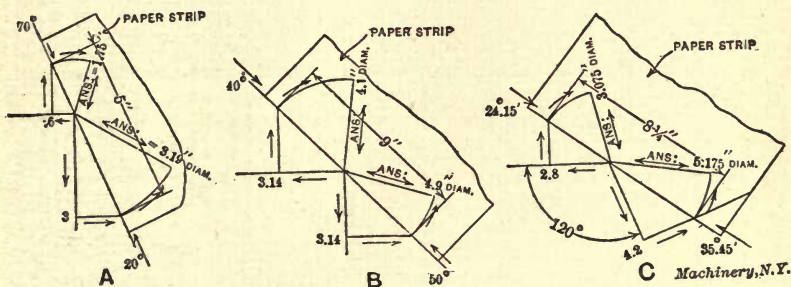


Fig 18. Separate Diagrams for the Solution of some of the Problems Presented

two numbers, when projected onto two spiral angle lines in a diagram made up as in Fig. 15, the sum of the angles of which equals 120 or 60 degrees, give two diameters whose sum equals the center distances multiplied by 2, or 8.25. In this case we cannot use both parts of the diagram Fig. 15, as it is made up for shafts at 90 degrees angle, and for this reason we must take the two readings from the same part of the diagram. The ratios 3 and 4.5 at 30 degrees give corresponding diameters of 3.5 and 5.2, the sum being 8.7. The ratios 2.8 and 4.2 giving 14 and 21 teeth at 25 and 35 degrees, respectively, have diameters of 3.1 and 5.15 (equals 8.25). From this we see that we must use 14 and 21 teeth and the ratios 2.8 and 4.2. The diameters and spiral angles can now be obtained graphically and more accurately in this manner:

Draw two radial lines, as shown at C in Fig. 18, at 120 degrees angle on a separate piece of paper, and lay off on these to same scale 2.8 and 4.2. From these points draw lines at right angles to the radial lines. It is now necessary to find the position of a line 8.25 inches long, terminating upon these lines, and passing through the center. A

The cutters used for milling spiral or helical gears are standard spur gear cutters, the number of a cutter and its pitch for a given case being defined by the angle (with axis) and normal pitch. This diagram gives the numbers of the cutters only, the pitch having been previously determined.

The selection of the cutter is fixed by the formula given in the lower right-hand corner of the diagram. The delimiting curves thereon were plotted by the formula, the area between the curves being the field of intersection of the combinations of angles and numbers of teeth covered by each designated cutter number.

For example, suppose the angle of the teeth of a gear is 37 degrees with its axis, and the number of teeth is 48. The point A, at which the horizontal line (representing the tooth number), and the vertical line (representing the angle) intersect, falls within the area marked "Cutter No. 2". Therefore, a No. 2 cutter is required to cut a 48-tooth spiral gear having the teeth at an angle of 37 degrees with its axis.

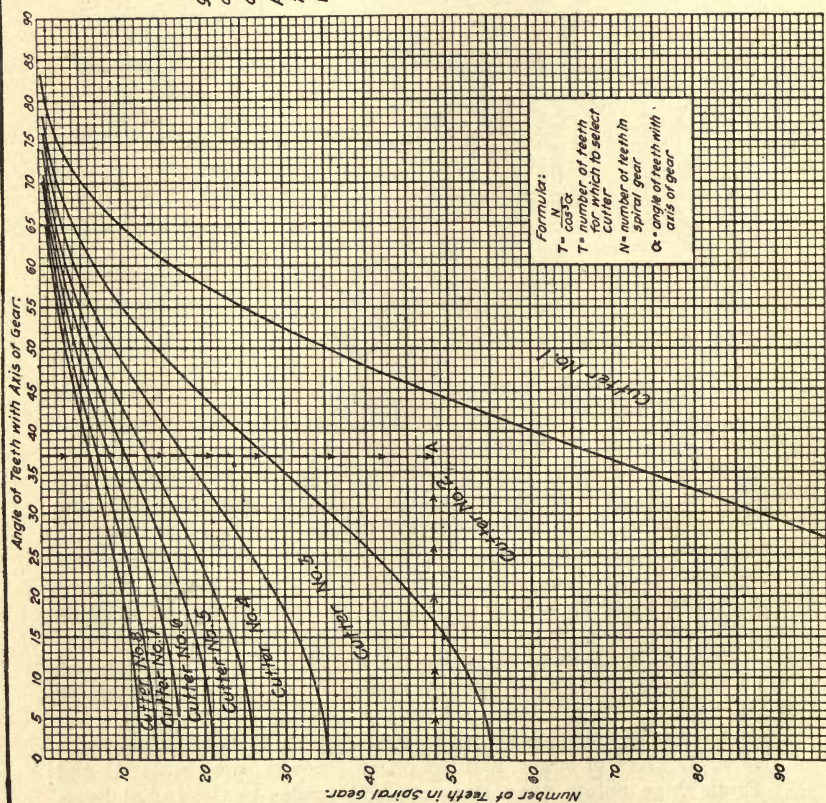


Fig. 19. Diagram for Determining Cutter to Use for Milling Spiral Gears

strip of paper is used in the same manner as before, and upon careful measuring of the respective distances from the center to the lines, one obtains the distances 3.075 and 5.175 inches, which represent the respective diameters, the sum being 8.25. The spiral angles are obtained by measuring or calculating as follows:

$$\begin{aligned}\cos \text{ spiral angle of first gear} &= 2.8 \times \frac{1}{3.075} = 0.910; \\ \text{spiral angle} &= 24 \text{ deg. } 15 \text{ min.}\end{aligned}$$

$$\begin{aligned}\cos \text{ spiral angle of second gear} &= 4.2 \times \frac{1}{5.175} = 0.812; \\ \text{spiral angle} &= 35 \text{ deg. } 45 \text{ min.}\end{aligned}$$

The above examples will show the careful student the manner of working out various problems as required, and if the directions are properly followed, this method will be found to be a great time-saver. It may be mentioned that it is advisable to keep the spiral angle as nearly equal in the two gears as possible in order to obtain the greatest efficiency of transmission. It should be noted that when diagrams of this type are to be used for practical calculation of spiral gears, they should be laid out in a much larger scale than is possible to show in these pages, and it would be advisable to lay out radial lines in Fig. 15 for every degree, and vertical and horizontal lines for every tenth of an inch, and circular arcs for equally fine subdivisions. The same is true of the diagrams in Figs. 16 and 17. In Fig. 16, horizontal lines should be laid out for every tenth of an inch, and vertical lines should be laid out for all whole numbers of teeth. In Fig. 17, the horizontal lines should be laid out for every tenth of an inch, vertical lines for at least every 0.2 of an inch, and radial lines for every degree. This diagram should also be laid out so that leads over 20 inches may be read off, as well as those below this figure.

In Fig. 19 is given a diagram for determining the cutter to use when milling the teeth of spiral gears. The instructions for the use of the diagram are given directly on the chart itself, so that no other explanation is necessary. This diagram was contributed to *MACHINERY* by Elmer G. Eberhardt, and appeared in the September, 1907, issue.

## CHAPTER III

### HERRINGBONE GEARS\*

The following information on herringbone gearing is abstracted from a paper by Mr. Percy C. Day, of Milwaukee, Wis., read before the meeting of the American Society of Mechanical Engineers, under the auspices of the sub-committee on machine shop practice, at New York, December 5-8, 1911. This abstract was published in *MACHINERY*, January, 1912.

That the helical principle in toothed gearing is ideal from a theoretical point of view is well known. From a practical standpoint, so called "herringbone" gears have, however, been less satisfactory than straight-cut spur gears, because, until recently, no method was devised for producing them with the requisite speed and accuracy. Within the last six years, however, a method has been developed, in England, to a high degree of perfection. Herringbone gears made by this method are called Wuest gears, after the inventor. The distinction between these gears and those of the ordinary herringbone type is that the teeth of the former, instead of joining at a common apex at the center of the face, are stepped half the pitch apart and do not meet at all. This arrangement of the teeth does not affect the action of the gears, but it facilitates their commercial production.

#### Action of Spur Gearing

The aim of all designers of gearing is to transmit rotary motion from one axis to another in a perfectly even manner without variation of angular velocity. Let us consider the action of a straight spur pinion driving a gear. There are three distinct phases of engagement:

First phase: The root of the pinion tooth engages the point of the gear tooth.

Second phase: The teeth are engaged near the pitch line.

Third phase: The point of the pinion tooth engages the root of the gear tooth.

Let us assume that the teeth are accurately cut to involute form, so that if the pinion moves with even angular velocity it will produce corresponding evenness of motion in the gear; and also that the pinion has sufficient teeth to allow the engagement of successive teeth to overlap. At the beginning of the first phase, while the load is carried near the point of the gear tooth, that tooth is subjected to a maximum bending stress along its whole length. The portion of the pinion tooth near the root is sliding over the outer portion of the gear tooth; that is to say, two metallic surfaces of small area are sliding under heavy compression.

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\**MACHINERY*, Engineering Edition, January, 1912.

The action during the second phase more nearly approaches ideal conditions. The teeth are engaged near their respective pitch lines and very little sliding takes place. During the third and final phase, the pinion tooth is subjected to a maximum bending stress, while the tooth surfaces again slide over each other, this time with the outer portion of the pinion tooth engaging the gear tooth near its root. The point to be noted is that while those portions of the mating teeth which are near the pitch lines transmit the load with rolling contact, those which are more remote have to transmit the same load with sliding contact. The inevitable result is that the points and roots of all the teeth tend to wear away more rapidly than the portions near the pitch lines.

It may be suggested that the sliding action can be eliminated by shortening the teeth so that they engage only the phase of rolling contact. This has been tried with a certain measure of success in the stub-toothed gear, but it cannot be carried far enough without curtailing the arc of contact so that continuity of engagement is lost.

#### Action of Herringbone Gears

Herringbone gears completely overcome all these difficulties, but only when they are accurately cut. If we take two exactly similar pinions with straight teeth and place them side by side on one shaft, with the teeth of one pinion set opposite the spaces of the other, then we have what is known as a stepped-tooth pinion. If this pinion is meshed with a composite gear made up in a similar manner, the action is modified so that there are always two phases of engagement taking place simultaneously. Such gears are commonly used for rolling mill work, because they stand up to heavy shocks better than the plain type. Still better action can be secured by assembling a number of narrow pinions with the last of the series one pitch in advance of the first and the others advanced by equal angular increments. As a practical proposition, however, gears made on these lines would be costly and difficult to produce.

The helical gear is the logical outcome of the stepped gear carried to its limit, and built up from infinitely thin laminations. Since the steps have merged into a helix, there must be a normal component of the tangential pressure on the teeth, producing end thrust on the shafts. To obviate end thrust, the helical teeth are made right-hand on one side and left-hand on the other. (See Fig. 20.) Such gears, with double helical teeth, are known as herringbone gears.

The fundamental principle of the action of herringbone teeth lies in the circumstance that *all phases of engagement take place simultaneously*. This holds good for every position of pinion and gear, provided only that the relationship between pitch, face width, and spiral angle is such as will insure a complete overlap of engagement. Since all phases of engagement occur together, it follows that the load is partly carried by tooth surfaces in sliding contact and partly by surfaces in rolling contact.

Those portions of the teeth farthest from the pitch line, which engage with sliding action, tend to wear away more rapidly than the portions nearest the pitch line. But the pitch line portion is always carrying part of the load, and the effect of wear on the ends of the teeth merely tends to throw more load on the center portions; in other words *there is a tendency to concentrate the load near the pitch lines*. The ends of the teeth, instead of wearing away to an ever-increasing extent from their original involute form, are relieved of some of the load from the moment that wear commences to take place. As soon as the load on these ends has been partially relieved and transferred to the middle portion, the wear becomes equalized all over the teeth and they do not tend to distort further from their original shape.

As the teeth keep their involute form, motion is transmitted from pinion to gear in an even manner, without jar, shock, or vibration. While herringbone teeth may not be intrinsically stronger than straight teeth, the elimination of shock renders them capable of transmitting heavier loads. Since all phases of engagement occur simultaneously, the transference of the load from one pinion tooth to the next takes place gradually instead of suddenly. This is the second principle of herringbone gearing, and may be termed *continuity of action*. In straight gears the continuity of action is a function of the number of teeth in the pinion. In herringbone gears continuity depends on the relationship between the face width and the number of teeth in the pinion. Pinions with as few as five teeth have been used with success by merely increasing the face width to suit such extreme conditions. This feature, which is peculiar to herringbone gears, has made practical the adoption of extremely high ratios of reduction hitherto considered impossible.

The third principle of herringbone gearing is that the bending stress on the teeth does not fluctuate from maximum to minimum as in straight gears, but remains always near the mean value. This feature is of special importance in rolling-mill driving and work of a similar nature.

To summarize the foregoing statements: The action of herringbone gears is continuous and smooth; there is no shock of transference from tooth to tooth; the teeth do not wear out of shape; the bending action of the load on the teeth is less than with straight gearing and does not fluctuate to anything like the same extent; the gears work silently and without vibration; back-lash is absent; friction and mechanical losses are reduced to a minimum; herringbone gears can be used for higher ratios and greater velocities than any other kind.

It has been explained that the teeth of the Wuest gears are so designed that those on the right- and left-hand sides of the gears are stepped half a space apart, and do not meet at a common apex at the center of the face, as in the usual type of herringbone gear. It has often been argued that the ordinary herringbone tooth is stronger than the Wuest tooth, because the latter lacks the support given by the junction of the teeth at the center. This argument would be sound if

gear teeth were ever stressed to anywhere near their breaking point. But it has been found in practice that considerations of wear so far outweigh those of mere breaking strength that a gear which is designed to give reasonable service will carry anywhere from ten to twenty times the working load without fracture. A point of vastly greater importance is that the stepped form will wear more evenly under extreme loads than the ordinary type. The reason for this is shown in Figs 20 and 21. The resultant tooth pressure is always normal to the teeth and tends to bend them apart. The stepped form offers a uniform resistance along its whole length, carrying the load from end to end (Fig. 20). The teeth of ordinary herringbone gears tend

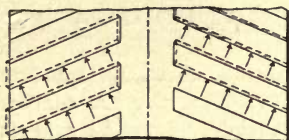


Fig. 20

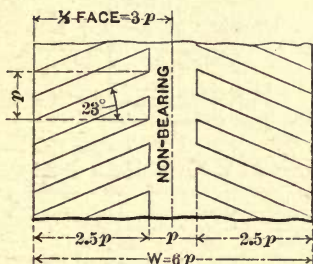


Fig. 22

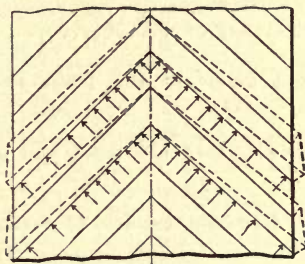


Fig. 21

Figs. 20 to 22. Diagrams showing Tooth Pressures and Angle Necessary for Continuity of Action

to separate more at the sides than near the supported center, causing the load to be concentrated toward the center (Fig. 21).

The standards which have been adopted for Wuest gears are the result of experience gained in Europe during the last six years. The spiral angle of the teeth is about 23 degrees with the axis. Since the nature of the action eliminates shock, it follows that the pitch required for given conditions will be much finer than would be chosen for spur gears. On the other hand, the face width will not be less, because there is as much necessity for wearing surface with one kind of tooth as with the other. Spur gears are usually made with a face width equal to three or four times the pitch. Herringbone gears may conveniently have a face width equal to six times the pitch, not because the width of this type need actually be greater, but by reason of the pitch being proportionately less.

Starting with a width equal to six times the pitch, and allowing one times the pitch as the non-bearing portion in the center, there remains

two and one-half times the pitch available for the teeth on each side. To insure continuity of engagement under all ordinary conditions, each tooth is inclined so as to cover an advance of one times the pitch within its length. The angle of 23 degrees satisfies this requirement (see Fig. 22).

The pressure angle which has been adopted for standard gears is 20 degrees. The teeth are shorter than the usual standards, because the high ratios used with these gears call for an average pinion diameter which is less than is used with straight spur gears for similar duty. The teeth are generated by hobs, and the short addendum combined with wide angle gives satisfactory tooth shapes, without undercutting of teeth on small pinions.

The dimensions proposed for an interchangeable system for these gears are as follows:

Tooth shape .....	Involute
Pressure angle .....	20 degrees
Spiral angle .....	23 degrees
	Number of teeth
Pitch diameter (20 teeth and over) =	$\frac{\text{D.P.}}{\text{Number of teeth} + 1.6}$
Blank diameter (20 teeth and over) =	$\frac{\text{D.P.}}{0.95 \times \text{Number of teeth} + 1}$
	0.95 × Number of teeth + 1
Pitch diameter (under 20 teeth) =	$\frac{\text{D.P.}}{0.95 \times \text{No. of teeth} + 2.6}$
	0.95 × No. of teeth + 2.6
Blank diameter (under 20 teeth) =	$\frac{\text{D.P.}}{0.8}$
	0.8
Addendum .....	$\frac{\text{D.P.}}{1.0}$
	1.0
Dedendum .....	$\frac{\text{D.P.}}{1.8}$
	1.8
Full depth .....	$\frac{\text{D.P.}}{1.6}$
	1.6
Working depth .....	$\frac{\text{D.P.}}{\text{D.P.}}$
	D.P.

Standard face width for gears with pinions of not less than 25 teeth, 6 times circular pitch; face widths for high-ratio gears with small pinions, 6 to 12 times circular pitch.

When a pinion of less than 20 teeth is used with a standard gear, the center distance must be slightly increased to suit the enlargement of the pinion. If it is desired to keep the center distance to the standard dimensions, the gear diameter may be reduced by the amount of the enlargement given to the pinion. For example: If a pinion of 10 teeth, 5 diametral pitch is to mesh with a gear of 90 teeth at 10-inch centers,

$$\text{Pitch diameter of pinion} = \frac{0.95 \times 10 + 1}{5} = 2.1 \text{ inches.}$$

Enlargement over standard pinion = 0.1 inch,

$$\text{Pitch diameter of standard gear} = \frac{90}{5} = 18.0 \text{ inches.}$$

Reduced pitch diameter of gear = 18.0 — 0.1 = 17.9 inches.

$$\text{Center distance} = \frac{17.9 + 2.1}{2} = 10 \text{ inches.}$$

### Power Transmitted by Herringbone Gears

The important factor in determining the proportions of the teeth is the relationship between pitch line velocity and the permissible

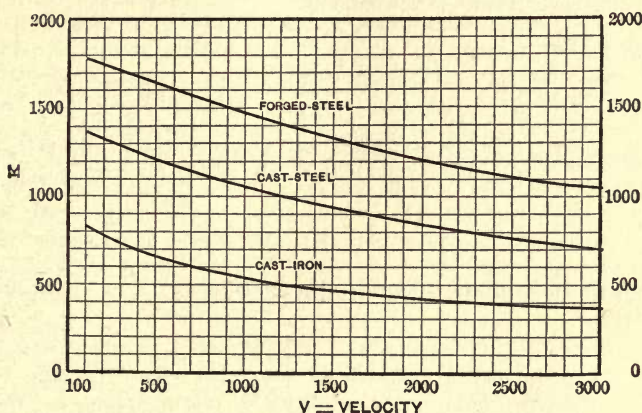


Fig 23. Shearing Stress with Relation to Pitch Line Velocity

specific tooth pressure; in other words, the total tooth pressure divided by the area of all the available simultaneous contact along the teeth. Theoretically, this contact has no area since it should consist of lines without breadth. Actually, an area exists, due to the elastic compression of the teeth in contact, in a similar way in which an area of contact exists between a car wheel and a rail. The area of contact is indeterminate, but the specific tooth pressure is proportional to the driving stress on the teeth.

In order to obtain a simple rule for finding the proper dimensions, the results of experience in the matter of safe working loads under given conditions have been reduced to a relationship between pitch line velocity and the shearing stress on the pitch line thickness of an imaginary straight tooth, assuming only one tooth in engagement at a time. The shearing stress is a measure of the specific tooth pressure, and the relationship referred to affords a convenient means of arriving at reliable dimensions. The curves, Fig. 23, give values of shearing stress  $K$  in pounds per square inch on the pitch line section of

an imaginary single tooth for corresponding pitch line velocities  $V$  in feet per minute. The values are entirely empirical, but they are based on the results of extended experience, and lead to dimensions which are safe and reliable. Different curves are given for different materials, and it is necessary to use that curve which corresponds to the lowest grade material of the combination. The dimensions of gears can be derived from the curves in the following manner:

H.P. = brake horsepower transmitted,

$N$  = revolutions per minute,

$D$  = pitch circle diameter, inches,

$p$  = circular pitch in inches (use nearest diametral pitch),

$W$  = total width of face, inches,

$V$  = pitch line velocity, feet per minute,

$P$  = total tooth pressure at pitch line, pounds,

$K$  = stress factor (from curve).

Then

$$V = \frac{\pi D N}{12} \qquad P = \frac{\text{H.P.} \times 33,000}{V} \qquad P = \frac{p W K}{2}$$

$$P = 3 p^2 K \left\{ \begin{array}{l} \text{in normal gears of moderate ratio, and face} \\ \text{width equivalent to six times the circ. pitch} \end{array} \right\}$$

$$p = \sqrt{\frac{P}{3 K}}$$

For high ratio gears take  $W = R p$  ( $R$  = ratio) up to maximum of  $W = 10 p$ .

$$p = \sqrt{\frac{2.5 P}{R K}}$$

In normal gears it is safe to aim at pitch line velocities between 1000 and 2000 feet per minute, with 1500 feet as a fair average. If the pinion is to be fixed to a motor shaft without external support, the diameter must be greater than when it can be supported on both sides. Cast iron is preferable to cast steel for gears of large diameters and moderate power, but the latter will be found more economical for high tooth pressures. Pinions are usually made from steel forgings of 0.40 to 0.50 per cent carbon. Soft pinions should never be used for herringbone gears.

There are two special cases where the ordinary methods of calculation should not be used. Rolling-mill gears are subjected to stresses which are so far in excess of the average working load that it is necessary to consider carefully the strength of the teeth in regard to possible overloads. Extra high velocity gears, such as are used for steam turbines, require additional wearing surface and are characterized by extreme width of face combined with abnormally fine pitch.

## CHAPTER IV

### CALCULATING GEARS FOR GENERATING SPIRALS ON HOBBING MACHINES\*

From time to time formulas have been developed for calculating the gears to be used for generating spiral gears. Those published in the past, however, have applied only to certain types of gear-hobbing machines. In the following a formula is given which is applicable to any type of gear-hobbing machine, and which is simpler to use than any formula so far published. In developing this formula, simple arithmetical expressions have been made use of, as far as possible, in order to make it especially useful to the practical man.

In order to clearly understand the use of any formula, it is necessary to know something of the principles involved. Fig. 24 shows a top view of a standard hobbing machine (the No. 3 Farwell) designed for cutting spur gears. Before dealing with the change gear ratios for spiral work, it will be well to have the methods for cutting spur gears firmly fixed in our minds. Assume the hob to be single threaded. It is evident that for each revolution of the hob, the gear being cut must move one tooth. Therefore, the hob revolves, for each revolution of the blank, as many times as there are teeth to be cut. To cut 44 teeth, we must gear the table to revolve once for every 44 revolutions of the hob.

The bevel gearing at *D*, Fig. 25, has a ratio of 3 to 1, the worm at *E* is double-threaded, and the worm-wheel *F* has 40 teeth. Hence the shaft *B* must revolve  $3 \times 44$  times for each revolution of the table, and the worm shaft *C* must revolve 20 times for each revolution of the table. Hence we have:

$$\frac{\text{Revolutions of } B}{\text{Revolutions of } C} = \frac{3 \times 44}{20}$$

Inverting this ratio to get the change gear ratio required to obtain this result, we have:

$$\frac{20}{3 \times 44} = \frac{\text{Product of No. of teeth in driving gears}}{\text{Product of No. of teeth in driven gears}}$$

In the following formulas, we will designate the product of the number of teeth in the driving gears *P*, and the product of the number of teeth in the driven gears *p*.

Should we use a double-threaded or triple-threaded hob, the gear we are cutting must revolve two or three teeth for each revolution of the hob; in other words, the speed of the table is increased directly as the number of threads on the hob, so we must multiply the number

\*MACHINERY, Engineering Edition, December, 1911.

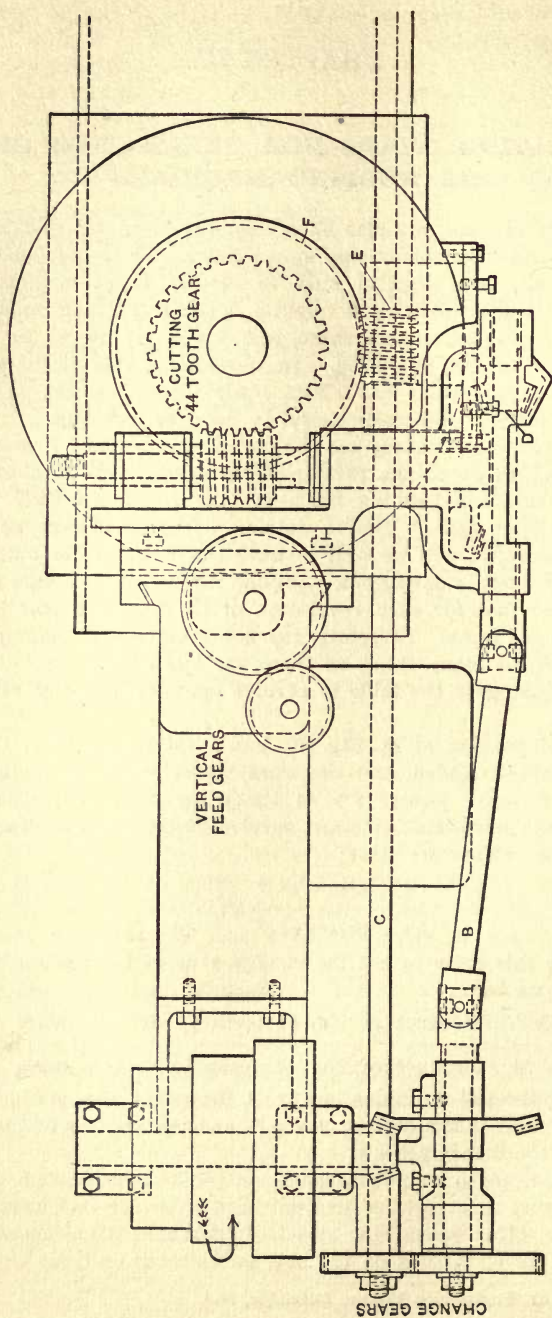


Fig. 24. Hob and Table Driving Arrangement of a No. 3 Farwell Gear-hobbing Machine

of teeth in the driving gears by the number of threads on the hob, giving us this formula:

$$\frac{20 \times \text{No. of threads on hob}}{3 \times \text{No. of teeth to be cut}} = \frac{P}{p}$$

A similar formula may be worked out in this way for any type of gear hobber.

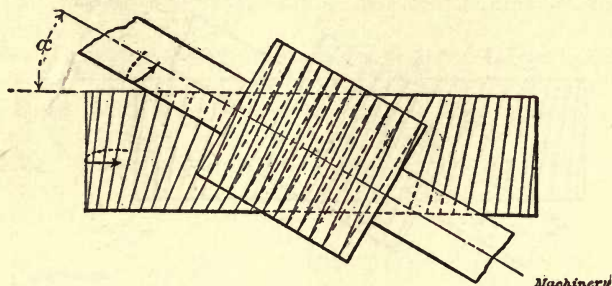


Fig. 25. Cutting a Right-hand Spiral Gear with a Right-hand Hob

#### Generating Spirals

For each revolution of the table, the head carrying the hob feeds down a certain distance across the face of the blank, this distance varying from 0.010 to 0.150 inch in common practice. To fully understand the following discussion, the action of the machine, as illustrated in Figs. 25 to 28, inclusive, should be noted. In Fig. 25 is shown the generation of a right-hand spiral gear with a right-hand hob; in Fig.

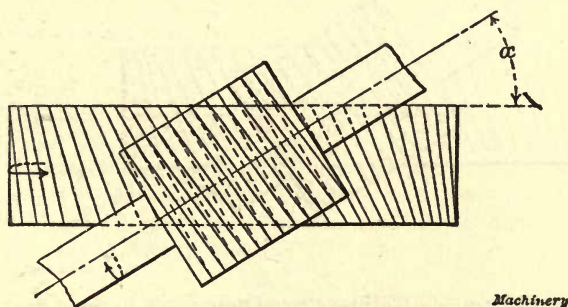


Fig. 26. Cutting a Left-hand Spiral Gear with a Right-hand Hob

26, a left-hand spiral gear with a right-hand hob; in Fig. 27, a left-hand spiral gear with a left-hand hob; and in Fig. 28, a right-hand spiral gear with a left-hand hob. In each of these illustrations the direction of rotation of the table is indicated by the arrow showing the direction of rotation of the gear being cut. The direction of rotation of the hob is also indicated by an arrow showing the direction of rotation of its shaft. In Figs. 25 and 27, where a gear is cut with

a hob of the same "hand," the angle  $\alpha$ , as indicated, equals the difference between the tooth angle and the thread angle of the hob. In Figs. 26 and 28, where the gear and the hob are of different "hand," the angle  $\alpha$  equals the sum of the tooth angle and the thread angle of the hob. After this preliminary introduction, we are ready to deal intelligently with the problem in hand.

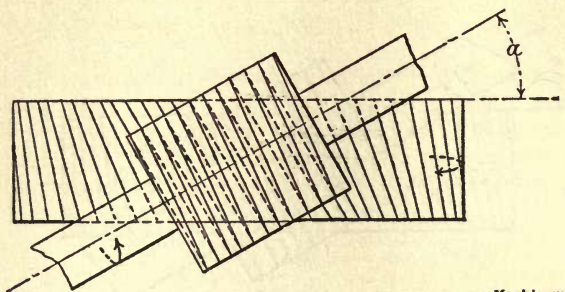


Fig. 27. Cutting a Left-hand Spiral Gear with a Left hand Hob

Assume the spiral gear shown in Fig. 29 to have sixty-four teeth. As indicated, the gear has a left-hand spiral and we will assume that it is cut with a left-hand hob. A single-threaded hob cutting a spiral gear would revolve sixty-four times for one revolution of the table; but since in this case the teeth are helical and the hob travels downward a certain distance, the position of the gear tooth must be advanced somewhat for every revolution with relation to the hob. In

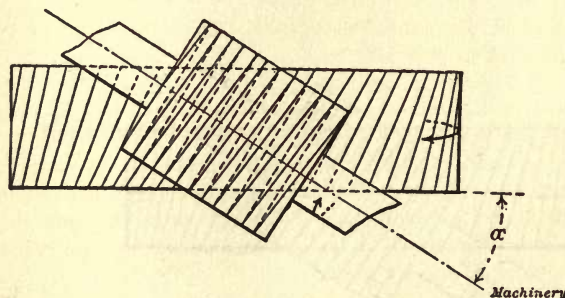


Fig. 28. Cutting a Right-hand Spiral Gear with a Left-hand Hob

other words, if the hob revolves sixty-four times, sixty-four teeth will have passed by, but the blank is not in the same position as at the beginning.

In Fig. 29,  $G$  represents the position of the hob axis at the beginning of the cut and  $H$  the position of the hob axis after the hob has made sixty-four revolutions. This shows that the blank must make more than one revolution in this case. If we were cutting a left-hand spiral gear with a right-hand hob, as shown in Fig. 26, the blank

would have to make less than one complete revolution for each sixty-four revolutions of the hob, the blank in this case being revolved in the opposite direction. It will thus be seen that when cutting a gear of the same "hand" as the hob, the table must revolve slightly faster than it would have to do when cutting a spur gear with the same number of teeth; but when the hob and the gear are of opposite "hand," the table must revolve more slowly than when cutting a spur gear. This has an important bearing upon the formula we are about to construct.

To gear the machine properly we must first find the ratio according to which the table is required to lag behind or lead ahead of its natural speed relative to the hob. In the first formula devised by the writer for the hobbing of spiral gears, the ratio was arrived at by considering the number of revolutions made by the hob, while the table

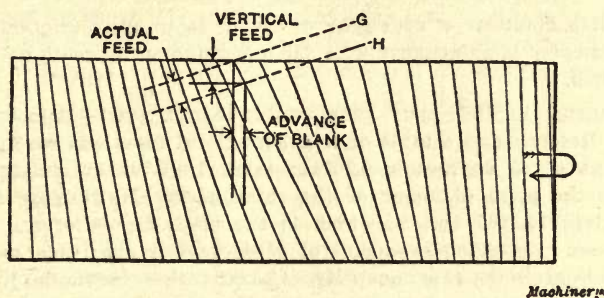


Fig. 29. Diagram showing Advance Required in Table Motion when cutting a Left-hand Spiral Gear with a Left-hand Hob

makes one full revolution. The formula thus constructed for the No. 1 Farwell gear-hobbing machine is:

$$\frac{30 \times \text{No. of threads on hob}}{\text{No. of teeth} \pm [(\text{feed} \times \tan. \text{ of angle}) \div \text{circ. pitch}]} = \frac{P}{p}$$

This applies only to one particular machine. A later formula designed for the No. 3 Farwell machine, as shown in Fig. 24, considers the number of table revolutions required while the hob revolves a sufficient number of times to represent one revolution of the table, if we were cutting a spur gear:

$$\frac{20 \pm \frac{20}{\text{Pitch circumference} \div (\text{feed} \times \tan \text{ of angle})}}{(3 \times \text{No. of Teeth}) \div \text{No. of threads on hob}} = \frac{P}{p}$$

Being called upon to derive another formula to be used for the new No. 3 Farwell universal hobbing machine, it occurred to the writer that a formula adapted to all hobbing machines would avoid much confusion. In the following is given the process by which such a formula was derived; the result is a simpler formula than any previously used.

The "lead" of a spiral gear is the axial length of the blank in which one spiral tooth makes a complete turn around the blank. Now, in hobbing a gear with a width of face exactly equal to the lead, it is evident that the blank must gain or lose one complete revolution as compared with the number of revolutions that would be made in cutting a spur gear with the same width of face and using the same feed per revolution of the blank. Assume that we wish to cut a 30-tooth, 10-pitch, right-hand spiral gear of 45-degree angle, using a single-threaded right-hand hob and feeding  $1/32$  inch across the face of the blank for each revolution of the blank.

The rule for finding the lead of a spiral gear is:

$$\text{Pitch circumference} \times \cot. \text{ of tooth angle} = \text{lead.}$$

To get the pitch circumference, we must first find the pitch diameter; the rule for finding this in a spiral gear is:

$$\text{Pitch diameter of spur gear} \div \cos. \text{ of tooth angle} = \text{pitch diameter of spiral gear with the same number of teeth and pitch.}$$

A 30-tooth, 10-pitch spur gear would have a pitch diameter of 3 inches. Referring to a table of trigonometrical functions we find that the cosine of 45 degrees is 0.70711; then,  $3 \div 0.70711 = 4.242$  inches, which is the pitch diameter of the spiral gear. Multiplying this by 3.1416 gives 13.3267 inches, which is the pitch circumference of the spiral gear. Since the cotangent of 45 degrees is exactly 1, multiplying by this gives the same quantity (13.3267 inches) as the lead.

The next step is to find how many times the blank must revolve while the hob feeds 13.3267 inches across its face. Since the feed is  $1/32$  inch (0.03125 for each revolution, we can divide by 0.03125 or multiply by 32 to get the number of revolutions. This gives 426.454 revolutions. The table has been traveling faster in relation to the hob than would be the case in cutting a spur gear with the same number of teeth; in fact, the table has gained exactly one revolution on the hob. In other words, the table speed in cutting this spiral gear is to the table speed in cutting an equivalent spur gear as 426.454 is to 425.454. From this we may construct the following formula:

$$\frac{\text{Lead} \div \text{feed}}{(\text{Lead} \div \text{feed}) - 1} = \frac{\text{required table revolutions}}{\text{normal table revolutions}}$$

For a gear of opposite "hand" from that of the hob the sign would be changed to + in this formula. Use the — sign only when gear and hob are of the same "hand."

By adding a 426-tooth gear to the drivers and a 425-tooth gear to the driven gears in the regular combination used to cut a 30-tooth spur gear, we would get approximately the desired ratio, but for greater accuracy we can carry the figures to a few decimal places and factor:

$$\frac{42645}{42545} = \frac{8529}{8509} = \frac{3 \times 2843}{67 \times 127}$$

But 2843 is a prime number. We, therefore, try

$\frac{4265}{4255} = \frac{853}{851}$ ; but 853 is a prime number. We therefore, try

$\frac{4264}{4254} = \frac{2132}{2127} = \frac{4 \times 533}{3 \times 709}$ , but 709 is a prime number. Hence we must

make another slight change and try again, remembering that whatever change is made in the numerator must be exactly duplicated in the denominator to maintain the ratio as nearly as possible. The dropping of all decimals would cause a very small error, but dropping them from one side only would cause a great error. We find upon trial that

$$\frac{426}{425} = \frac{2 \times 3 \times 71}{5 \times 5 \times 17}$$

Multiplying this with the change-gear combination ordinarily used to cut spur gears with 30 teeth, we have the gear combination required for any gear-hobbing machine used for cutting this gear. Thus, on the No. 3 Farwell universal hobbing machine, the spur gear ratio for cutting 30 teeth is  $\frac{30}{60}$ , which multiplied by  $\frac{2 \times 3 \times 71}{5 \times 5 \times 17}$  gives  $\frac{3 \times 71}{5 \times 5 \times 17}$ , and arranging this ratio in convenient gear sizes, we have:

$$\frac{24 \times 71}{40 \times 85} = \frac{\text{Product of teeth of driving gears}}{\text{Product of teeth of driven gears}}$$

It will be noted that the last operation before factoring was to divide by the feed. Should prime numbers be encountered repeatedly in trying to factor, it is possible to get altogether new figures to work with, by making a slight change in the feed and dividing into the lead again.

Having found the gears, set the feed for exactly  $1/32$  inch per revolution, see that the table is revolving in the right direction, and tilt the hob spindle to bring the *thread* angle to 45 degrees and the machine is ready for business.

#### Recapitulation and General Remarks

The general formula for gearing any hobbing machine for generating spiral gears is thus:

$$\frac{L \div F}{(L \div F) \pm 1} \times \frac{P}{p} = \frac{S}{s}$$

in which

$L$  = lead of spiral,

$F$  = feed per revolution,

$P$  = product of driving gears for cutting spur gears with same number of teeth,

$p$  = product of driven gears for cutting spur gears with same number of teeth,

$S$  = product of driving gears for cutting spiral gears,

$s$  = product of driven gears for cutting spiral gears.

Use + sign when gear and hob are of opposite "hand," and — sign when they are of the same "hand."

In cutting teeth at large angles it is desirable to have the hob the same hand as the gear, so that the direction of the cut will come against the movement of the blank, but at ordinary angles one hob will cut both right- and left-hand gears.

The actual feed of the cutter depends upon the angle of the teeth as well as on the vertical movement of the hob. This is obtained by dividing the vertical feed by the cosine of the tooth angle; thus:

$$\frac{0.03125}{0.70711} = 0.043 \text{ inch actual feed.}$$

The last computation need not be made except to see that we are not figuring on too heavy a cut, as it has nothing to do with the gearing of the hobbing machine. In setting up a hobbing machine for spiral gears, care should be taken to see that the vertical feed does not trip until the machine has been stopped or the hob has fed down clear of the finished gear. Should the feed stop while the hob is still in mesh with the gear and revolving at the ratio required to generate a spiral, the hob will cut into the teeth and spoil the gear.

Should the thread angle of the hob be exactly equal to the tooth angle of the spiral gear, and both hob and gear be the same "hand," the axis of the hob spindle will be at right angles to the axis of the gear. This is in conformity with the rule that when hob and gear are of the same "hand," the hob spindle is set at the tooth angle minus the thread angle of the hob. In cutting a spiral gear to take the place of a worm-wheel, it is possible to use the same hob that was used in cutting the worm-wheel. This would be a case where it is not necessary to tilt the hob spindle. Sometimes multiple-threaded hobs are used in order to make the thread angle approximately equal to the tooth angle, when it is desired to cut spiral gears with machines on which the hob spindle swivels through only a small angle.

## CHAPTER V

### THE SETTING OF THE TABLE WHEN MILLING SPIRAL GEARS\*

It has been frequently stated that the most suitable angle (and the one most likely to produce the best results) at which to set the table of the milling machine when milling spiral gears, is that corresponding either to the diameter of the gear measured at the bottom of the space, or to the diameter measured at the working depth. The reason invariably adduced for this is that, if the angle chosen is the angle of the spiral measured on the pitch cylinder of the gear, an undue amount of undercutting, and therefore weakening, of the teeth will

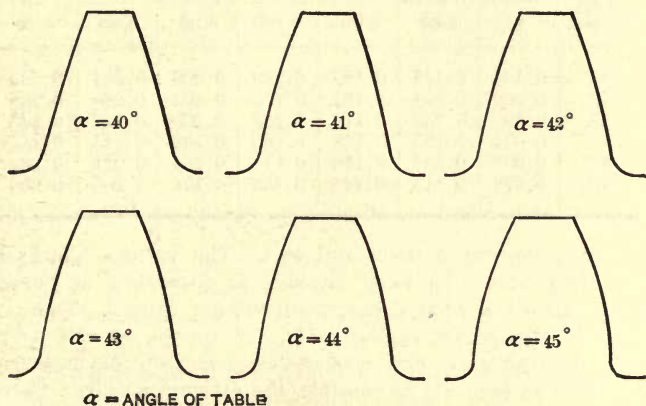


Fig. 30. Shapes of Teeth obtained by Setting the Table at Different Angles, Cutter and Lead remaining the same

occur, owing to an excessive amount of interference with the sides of the teeth on the part of the cutter; and that, therefore, a somewhat smaller angle should be selected to reduce these effects.

To determine whether there was, practically, anything in this idea or not, some experiments were recently made on a spiral gear, the immediate object of the experiments being to find out what the effect of altering the angle of setting of the milling machine table was upon the shape of the tooth cut.

The experiments were made upon a cast-iron gear, with a pitch diameter of 4.242 inches, and designed for 24 teeth, the diametral pitch (corresponding to the normal circular pitch) being 8. The

correct cutter to use was determined by the formula  $N_e = \frac{N}{\cos^3 \alpha}$ ,

\*MACHINERY, Engineering Edition, June, 1911.

this cutter being No. 3 in each of the cases dealt with. The experiments consisted of cutting six teeth in the gear blank, all being of the same depth, the angle of setting of the table of the milling machine being different in each of the six cases. The spiral angle measured on the pitch cylinder was 45 degrees, the lead of the spiral being 13.32 inches, for which the gears of the spiral dividing-head were arranged. The six spirals chosen were at angles of 45, 44, 43, 42, 41, and 40 degrees, each tooth being formed by two cuts at one angle, the lead of the spiral remaining the same throughout the series of tests. It should be here noted that 43 degrees is the angle which corresponds to the diameter measured at the bottom of the space.

The profiles of the teeth taken as sections normal to the spiral on the pitch surface are indicated in Fig. 30, the profiles being drawn ac-

TABLE OF OBSERVED TOOTH DIMENSIONS

Angle of Table Setting, Degrees	Width of Tooth at a Depth of Inches						
	0	0.050	0.100	0.125	0.150	0.200	0.250
45	0.104	0.145	0.185	0.200	0.208	0.221	0.239
44	0.102	0.144	0.181	0.195	0.202	0.220	0.238
43	0.099	0.142	0.176	0.188	0.200	0.218	0.236
42	0.094	0.135	0.168	0.180	0.196	0.215	0.234
41	0.087	0.128	0.158	0.171	0.185	0.211	0.232
40	0.078	0.115	0.146	0.158	0.171	0.205	0.230

curately to scale—three times full size. The various widths of the teeth at different depths were obtained as accurately as possible by means of a Brown & Sharpe gear-tooth vernier caliper. These widths are given in the accompanying table. Of course, it will be readily seen that although great care was exercised in securing measurements that would be as accurate as possible, the dimensions given above may be incorrect by about one or two thousandths inch, but not more.

In regard to the shapes of the teeth, it will be noticed that the 45-degree tooth is slightly undercut at the root, while the other teeth do not show any undercutting whatever. The undercutting referred to in the 45-degree tooth amounts to a reduction in width below the widest part of the tooth of about 0.010 inch.

The deductions drawn from the results of these tests are:

1. That the practice of setting the table at an angle less than the spiral or helix angle measured on the pitch surface is justified; though this angle should not be less than the spiral or helix angle measured at the bottom of the tooth.

2. That a cutter for a larger number of teeth than that given by the formula  $N_e = \frac{N}{\cos^2 a}$  should be employed, in order to counteract the flattening and widening effect of the cutter with an angle as indicated above.



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